Automatic Determination of Optimal Regularization Parameter in Rational Polynomial Coefficients Derivation

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2012. 12. 14

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Outline

Issue & Objective

- Increasing demand for accurate spatial information, massive archives of ground information are provided by imagery from various sensor (IKONOS, GeoEye etc.)
- To establish the functional relationship between image space and the ground space, sensor models are required
- There are two types of sensor model. One is rigorous sensor model and the other is generalized sensor model
- Rigorous sensor model – producing high accuracy but it is complicated, and most vendors kept confidential
- Therefore, most satellite vendors have adopted generalized sensor model

Issue & Objective

- The rational functional model(RFM) is one of the most popular generalized sensor model (The polynomial coefficients in RFM are called RPC(Rational Polynomial Coefficients))

- Comparing with the rigorous sensor model and RFM, the RFM would be over parameterized, so we can encounter numerical instability problem during RPC calculation

- To solve such problem, RP(Regularization Parameter) is used

- In many cases, RP is determined by heuristic (trial method)

- In this paper, we deals with automatic determination of optimal RP in RPC derivation

Former research for determining optimal RP

- L-curve & U-curve method:
  - most effective when the problem is well conditioned with high noise level
- OCV method:
  - provides more reasonable amount of regularization for ill conditioned with high noise level
- Ridge tracing method:
  - is independent of condition number and noise, but determined heuristic (trial method)
- In this paper, we introduce a numerical approach for automatic determination of RP in Ridge tracing method

RFM for ground and image coordinates

The RFM relates ground (X, Y, Z) coordinates to image coordinates (r, c) in the form of rational functions that are ratio of polynomials. For the ground to image transformation, the defined ratio of polynomials have following form

$$r_n = \frac{p_1(X_n, Y_n, Z_n)}{p_2(X_n, Y_n, Z_n)} = \frac{\sum_{i=0}^{m1}\sum_{j=0}^{m2}\sum_{k=0}^{m3} a_{ijk} X_n^{\,i} Y_n^{\,j} Z_n^{\,k}}{\sum_{i=0}^{n1}\sum_{j=0}^{n2}\sum_{k=0}^{n3} b_{ijk} X_n^{\,i} Y_n^{\,j} Z_n^{\,k}}$$

$$c_n = \frac{p_3(X_n, Y_n, Z_n)}{p_4(X_n, Y_n, Z_n)} = \frac{\sum_{i=0}^{m1}\sum_{j=0}^{m2}\sum_{k=0}^{m3} c_{ijk} X_n^{\,i} Y_n^{\,j} Z_n^{\,k}}{\sum_{i=0}^{n1}\sum_{j=0}^{n2}\sum_{k=0}^{n3} d_{ijk} X_n^{\,i} Y_n^{\,j} Z_n^{\,k}}$$

$$\begin{aligned} P_1 = {} & a_0 + a_1X + a_2Y + a_3Z + a_4XY + a_5XZ + a_6YZ + a_7X^2 + a_8Y^2 + a_9Z^2 + a_{10}XYZ \\ & + a_{11}X^3 + a_{12}XY^2 + a_{13}XZ^2 + a_{14}X^2Y + a_{15}Y^3 + a_{16}YZ^2 + a_{17}X^2Z + a_{18}Y^2Z + a_{19}Z^3 \end{aligned}$$

$$\begin{aligned} P_3 = {} & c_0 + c_1X + c_2Y + c_3Z + c_4XY + c_5XZ + c_6YZ + c_7X^2 + c_8Y^2 + c_9Z^2 + c_{10}XYZ \\ & + c_{11}X^3 + c_{12}XY^2 + c_{13}XZ^2 + c_{14}X^2Y + c_{15}Y^3 + c_{16}YZ^2 + c_{17}X^2Z + c_{18}Y^2Z + c_{19}Z^3 \end{aligned}$$

$$\begin{aligned} P_2 = {} & b_0 + b_1X + b_2Y + b_3Z + b_4XY + b_5XZ + b_6YZ + b_7X^2 + b_8Y^2 + b_9Z^2 + b_{10}XYZ \\ & + b_{11}X^3 + b_{12}XY^2 + b_{13}XZ^2 + b_{14}X^2Y + b_{15}Y^3 + b_{16}YZ^2 + b_{17}X^2Z + b_{18}Y^2Z + b_{19}Z^3 \end{aligned}$$

$$\begin{aligned} P_4 = {} & d_0 + d_1X + d_2Y + d_3Z + d_4XY + d_5XZ + d_6YZ + d_7X^2 + d_8Y^2 + d_9Z^2 + d_{10}XYZ \\ & + d_{11}X^3 + d_{12}XY^2 + d_{13}XZ^2 + d_{14}X^2Y + d_{15}Y^3 + d_{16}YZ^2 + d_{17}X^2Z + d_{18}Y^2Z + d_{19}Z^3 \end{aligned}$$

LS and ill-condition problem

To obtain the RPC(a_{ijk}, b_{ijk}, c_{ijk}, d_{ijk}), LS solution is used. Because RPC in RFM are highly correlated between coefficients, B matrix is usually ill-conditioned, and matrix N could become singular. Therefore, iterative solution cannot be converged. To tackle the ill-conditioned problem, RP technique is applied

Nonlinear cond. eq $F(l,x)=0$

linearization $v + B\Delta = f$

Partial derivation of F $B=[F(l,x^0)]$ $f = -[F(l,x^0)]$

Normal eq. $N = B^T WB$ → $N' = B^T WB + \lambda^2 I$

Correction mat.. $\Delta = N^{-1}(B^T Wf - f_x)$ → $\Delta' = N'^{-1}(B^T Wf - f_x)$

Parameters $\hat{x} = x^0 + \Delta$ → $\hat{x}' = x^0 + \Delta'$

RMSE function

Determining RP is not a trivial problem. RP should be large enough to make B matrix having small condition number, and should be small enough to minimize the error.

To solve the problem, RMSE function is proposed. $r_{\lambda i}$ is image coordinate obtained from specific RP, and r_{rigi} is image coordinate obtained from known ground coordinates and rigorous sensor model.

$$F_{RMSE}(\lambda) = \frac{\sum_{i=1}^{N_{CK}} \sqrt{[(r_{\lambda i} - r_{rigi})^2 + (c_{\lambda i} - c_{rigi})^2]}}{N_{CK}}$$

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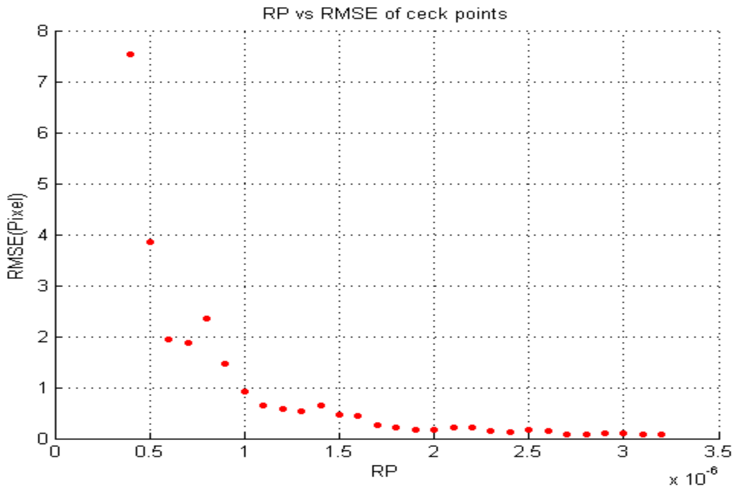
Optimal RP

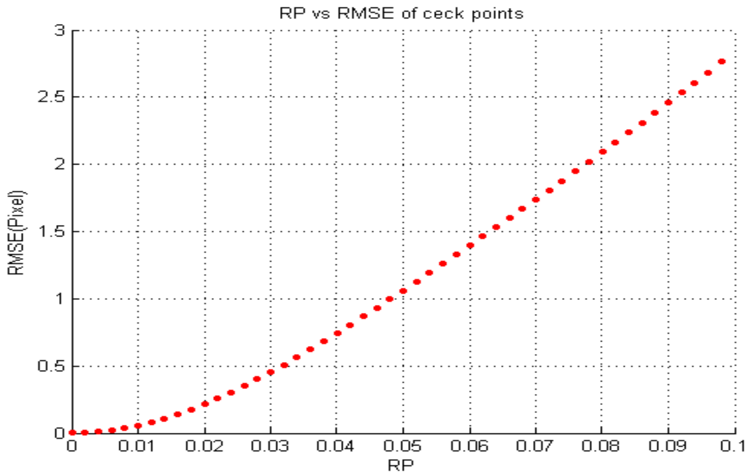
RMSE graph is like as follows. With increasing the RP, RMSE is decreasing, being flat, and increasing. With this kind of graph tendency, we can easily calculate the RP producing smallest RMSE in the graph by using numerical approach.

$$F_{RMSE}(\lambda) = \frac{\sum_{i=1}^{N_{CK}} \sqrt{[(r_{\lambda i} - r_{rigi})^2 + (c_{\lambda i} - c_{rigi})^2]}}{N_{CK}}$$

from 4x10^{-7} to 3.2x10^{-5}



from 4x10^{-5} to 1x10^{-1}



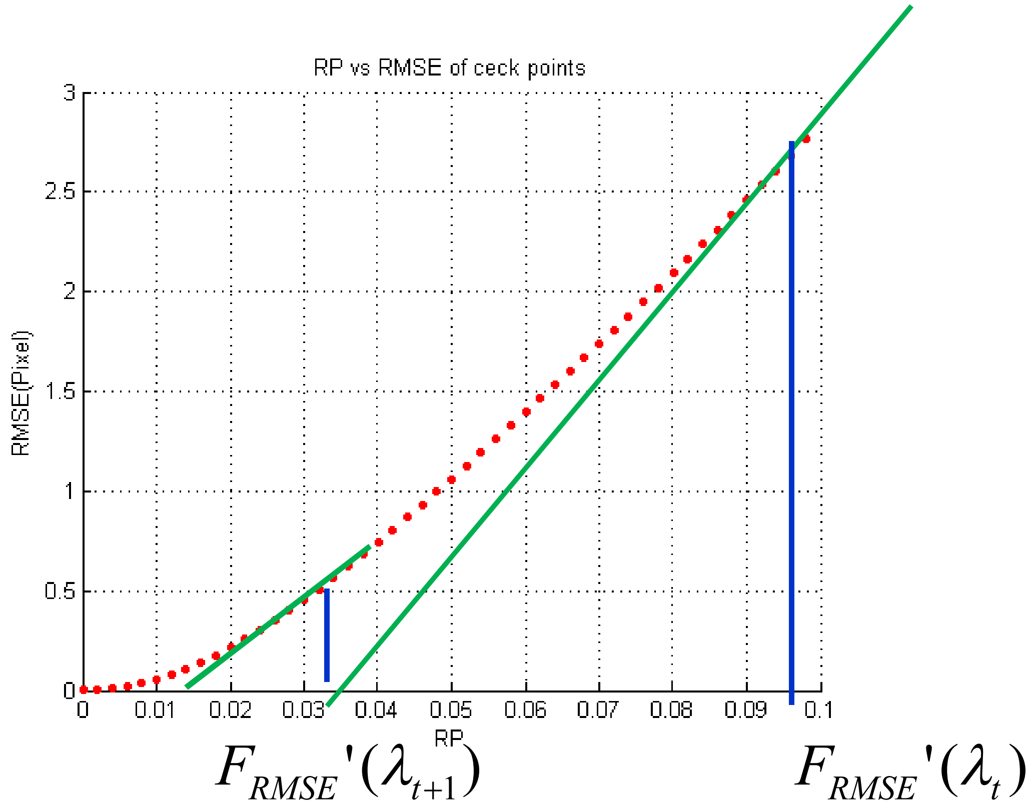
Optimal RP

Solution is finding first derivatives of RMSE being near to zero. The Euler' s root finding method simply find the point of which RMSE value is smallest. By using this, we can obtain the RP, of which first derivatives is significantly small.

$$F_{RMSE}(\lambda) = \frac{\sum_{i=1}^{N_{CK}} \sqrt{[(r_{\lambda i} - r_{rigi})^2 + (c_{\lambda i} - c_{rigi})^2]}}{N_{CK}}$$

$$\frac{F_{RMSE}(\lambda)}{\partial \lambda} = \frac{F_{RMSE}(\lambda + \Delta\lambda) - F_{RMSE}(\lambda)}{\Delta \lambda}$$

$$\lambda_{t+1} = \lambda_t - h \frac{F_{RMSE}(\lambda_t)}{\partial \lambda}$$

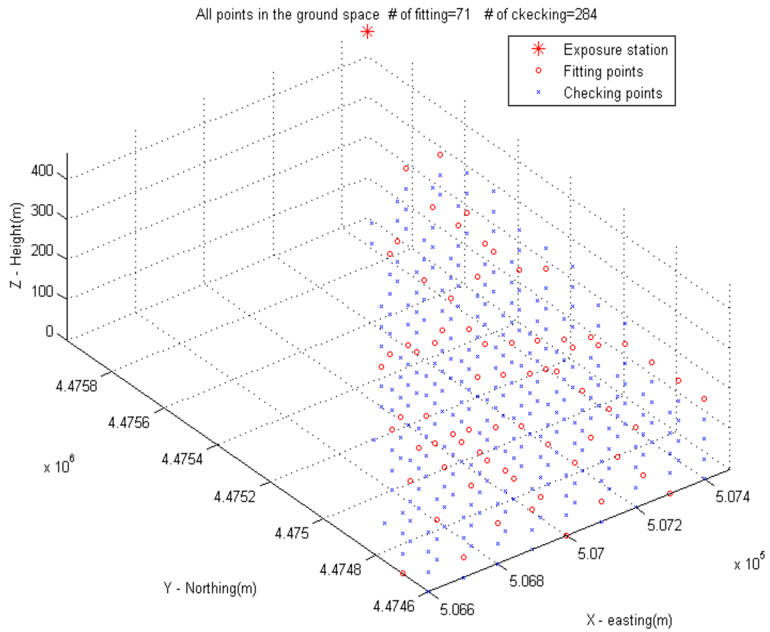


3D Object Grid in the Ground Space-1

A 3D object grid is established. The relief range is from 0-400 meter. The intervals of grid of easting, northing, and height are 100-100-50 meter.

Tactical image

Used points in the ground space (X-Y-Z)



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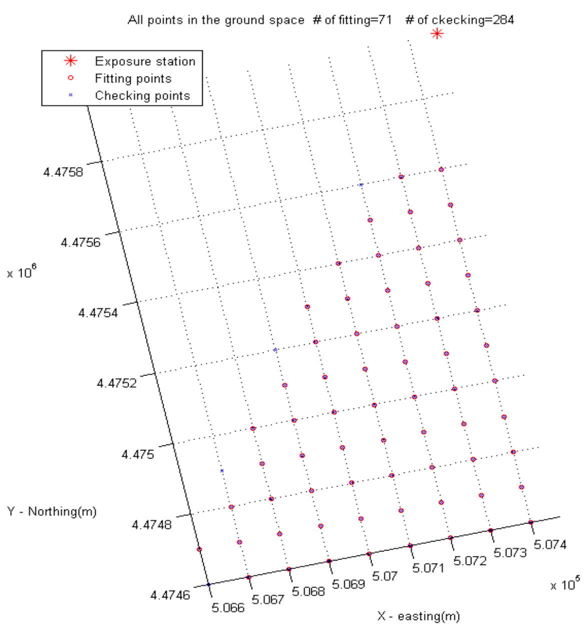
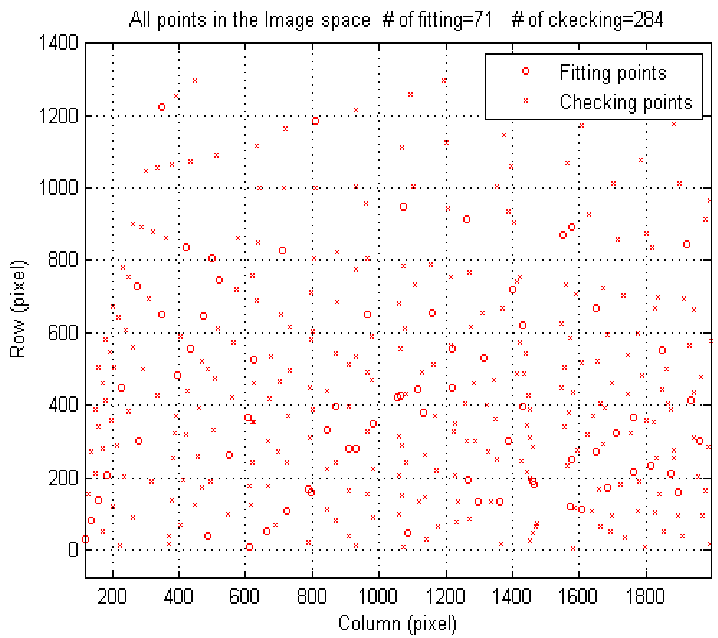
3D Object Grid in the Ground Space-2

A 3D object grid is established. The relief range is from 0-400 meter. The intervals of grid of easting, northing, and height are 100-100-50 meter.

Used points in the ground space (X-Y)



Used points in the image space



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Used Parameters for Rigorous Sensor Model

Rotation angle ω	-1.18445382 radian
Rotation angle φ	0.30607712 radian
Rotation angle κ	3.03356237 radian
Focal length f	3310.0 pixel
Coordinates of principal point (x_a, y_a)	(45.0,50.0)pixel
Exposure station X_L	507,471.474 meter
Exposure station Y_L	4,475,970.091 meter
Exposure station Z_L	463.380 meter

Results

$$F_{RMSE}(\lambda) = \frac{\sum_{i=1}^{N_{CK}} \sqrt{[(r_{\lambda i} - r_{rigi})^2 + (c_{\lambda i} - c_{rigi})^2]}}{N_{CK}}$$

$$\frac{F_{RMSE}(\lambda)}{\partial \lambda} = \frac{F_{RMSE}(\lambda + \Delta\lambda) - F_{RMSE}(\lambda)}{\Delta \lambda}$$

$$\lambda_{t+1} = \lambda_t - h \frac{F_{RMSE}(\lambda_t)}{\partial \lambda}$$

- Optimal RP is calculated
- Initial RP is set to 1×10^{-1}
- The step size h is set to 1×10^{-3}
- 1×10^{-5} for $\Delta \lambda$ is used
- With the experiment, determined RP is 2.024×10^{-4}, and RMSE for RP is 0.1612 pixel

Conclusion

- This paper deals with automatic determination of RP in RPC derivation
- We propose RMSE function for RP
- With Euler's root finding method, optimal RP can be automatically determined
- Experiment with simulated situation shows the effectiveness of the algorithm
- Our approach is independent of ill-conditioned or well conditioned, unlike L-curve and U-curve method
- Another advantage of proposed algorithm is that RP can be determined at the target accuracy with simple calculation

Thank You

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