3D SPATIAL RELATIONSHIPS MODEL: A USEFUL CONCEPT FOR 3D CADASTRE?

ROLAND BILLEN[1] AND SIYKA ZLATANOVA[2]

[1] University of Liège
Belgium
[2] Delft University of Technology
The Netherlands

ABSTRACT

This paper contains some reflections about using 3D topology, and more generally some 3D concepts related to 3D cadastre. Firstly, we develop some conceptual views about 3D modelling and more specifically about the definition of 3D spatial objects. Secondly, we present a new framework for representing spatial relationships. The framework has different "complexity" levels and allows mixing simple and complex spatial relationships. Therefore, it is possible to consider different levels when querying spatial objects. Finally, some examples are presented in R^3 that demonstrate the applicability of the framework for cadastral objects.

INTRODUCTION - THE EVOLUTION TO 3D

For many years, acquisition techniques and computational processes evolve continually and the practical limitations of the use of 3D information decrease. But in most of the cases and especially in urban contexts, the evolution to real 3D geo-objects is rather slow. This could be explained by a strong inhibitor factor, i.e. the inheritance of 2D way of thinking. The primary reflex when upgrading a 2D model, for example the cadastral model, may be to keep the 2D object's definition and add some 3D extensions. Even if the result is satisfactory, the approach is incomplete and limitative. The opportunity of working with 3D data allows us to consider the 3D world where many objects can significantly evolve. If an object has a new definition strongly related to 3D (note that it can be therefore seen like a new object), the use of a 3D model will be imperative by itself. To reach this objective, adequate abstractions and manipulation tools should be used. Among all, support of the 3D spatial analysis is most critical. In this order of ideas, we will present a new 3D conceptual model (the Dimensional Model) for describing real world and a new framework for representing spatial relationships in R^3.

3D OBJECTS OF INTEREST IN URBAN AREAS

Traditionally, the objects of interest in a GIS are considered spatial objects, i.e. objects that have thematic and geometric characteristics. Consequently, the word is about 3D GIS while the objects are geometrically represented in three dimensions. Several extended studies have been done about the 3D object of interest in urban environment. The common understanding is that the most important 3D real objects in urban areas are buildings and terrain represented as TIN (*Grün and Dan 1997, Leberl and Gruber 1996, Tempfli 1998*). *Fuch 1996* presents a study on real objects of interest for 3D city models. The investigations in five groups of objects (i.e. buildings, vegetation, traffic network, public utilities and telecommunications) have clearly shown the prevalent usage of (need for) buildings, traffic network and vegetation. *Razinger and Gleixner 1995* present a virtual model of a square in Graz (created upon municipal request), containing buildings, traffic network, lamp-posts and trees. *Dahany 1997* suggests three groups of objects to be considered: terrain, vegetation and built form. Clearly, most of the authors address real objects with spatial extent. Operational data needed for urban planning and especially cadastre, however, goes often far beyond the real objects of interest discussed so far. For example, cadastral offices maintain juridical boundaries and legal status of the real estate, i.e. items that cannot be classified as 3D spatial objects. *Zlatanova 2000* proposes objects as people, companies, taxes, etc. to be included in the scope of objects organised in a GIS. Four basic groups to distinguishing real objects are introduced, i.e. juridical objects (e.g. individuals, institutions, companies), topographic objects (e.g. buildings, streets, utilities), fictional objects (e.g. administrative boundaries) and abstract objects (e.g. taxes, deeds, incomes). Since all the objects have semantic characteristics, geometric characteristics of real objects are the leading criterion of the grouping. There are objects with either: 1) non-complete geometric characteristics (i.e. only location); 2) complete geometric characteristics and existence in the real world; 3) complete geometric characteristics and fictive existence; and 4) without geometric characteristics.

According to this classification, the 3D topographic objects are basically the 3D spatial objects currently maintained (or intended for maintenance) in a variety of information systems. The need of 3D fictional objects is usually not that transparent. While it seems normal to evolve from a 2D representation of building to a 3D representation (because this is the reality), this is not the case for fictional object (municipality unit, statistical unit, or other fictional phenomena).

The challenge of 3D GIS is to support analysis between all different types of real objects. If 3D GIS incorporates only 3D topographic objects and no 3D fictional objects, some analysis would be simplified or even truncated. Such simplification may also have the effect of a strong brake to the evolution of 3D GIS. Therefore, in this paper, we will consider both topographic and fictional objects as a part of our spatial relationship model.

THE DIMENSIONAL MODEL

The development of a mathematical theory to categorise relations among spatial objects has been identified in early 80-ties as an essential task to overcome the diversity and incompleteness of spatial relationships realised in different information systems. The intensive research in this area has led to the development of a framework based upon set theory and general topology principles and notions (see *Pullar and Egenhofer 1988*). The framework utilises the fundamental notions of general topology for topological primitives to investigate the interactions of the spatial objects. The basic criterion to distinguish between different relations is the detection of empty and non-empty intersections between topological primitives. Depending on the number of the topological primitives considered, two intersection models were presented in the literature. The first idea is to investigate the intersection of interiors and boundaries of two objects. This results in $2^4=16$ relations between two objects. Apparently, many relations cannot be distinguished on the basis of only two topological primitives. Therefore the evaluation of the exterior is adopted (the 9-intersection model, *Egenhofer and Herrring 1990*). The number of detectable relations between two objects thus increases to $2^9=512$. The criticism is mostly about the fact that not all the relations are possible in reality, the intersections are not further investigated, and many object intersections are topologically equivalent.

A slightly different approach is followed by Clementi et al 1993. Again, the three topological primitives are used but first the type of intersection is clarified and then a detailed evaluation of all the cells composing an object is performed. The approach claims a detection of a larger number of relations, however, at a high computational price.

This section presents a new framework, i.e. *the Dimensional Model (DM)* for describing real world and a new framework for representing spatial relationships in R3. The basic role of this model was to respond to the limitation of the 9-intersection model and more precisely the topological equivalence (*Egenhofer and Franzosa 1994, Zlatanova 2000*).

Prior giving the definitions, we will mention few key issues. First, the model is based on the 2D CONGOO formalism presented by Pantazis et al 1996 and extended to cope with 3D objects and spatial relationships. Second, the spatial object in this model is composed of different smaller elements named *dimensional elements*. Third, the conceptualisation of the spatial relationships (considered of superior importance compare to objects) focus on only objects with defined *limits* (will be explained later). The following subsections are organised as follows: first, the key definitions for representing objects of real world are presented and then the framework for describing spatial relationships is discussed.

Dimensional elements

This section presents the basic rules of the Dimensional Model. More details, i.e. the mathematical demonstrations of these rules for convex objects, will be given in further works. The starting definition of the model follows:

Definition 1: A ***Dimensional*** *element (denoted by D element) is a portion of space, which is related to a particular* ***dimensional frame of reference****. It is said* α*-dimensional when totally defined by* α*-dimensional frames of reference.*

Frame of reference denotes the dimension of the space (or sub-space) in which the dimensional element is embedded. Figure 1 shows a 1D element plunged into a 3D frame of reference (R^3). Each point of this element can be expressed also by one coordinate using the associated 1D frame of reference. Another example for a 2D element is given in Figure 2.

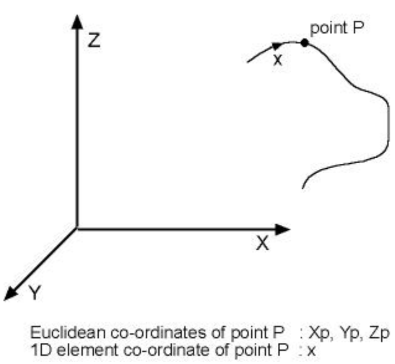


Figure 1: 1D element embedded in R^3.

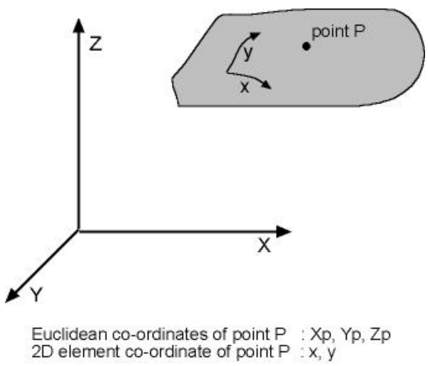


Figure 2: 2D element embedded in R^3.

By transitivity, one can say that a n-dimensional portion of space is denoted nD element. A n-dimensional frame of reference may contain nD elements, (n-1)D elements down to 0D elements. In the 3D Euclidean space (R^3), four types of dimensional elements are allowed, i.e. 0D, 1D, 2D and 3D elements.

Definition 2: A α-dimensional element has an ***extension*** *and may have a* ***limit****. The extension is the whole sub-space of α-dimension of the element, and the limit is the whole sub-space of 0 dimension to α-1 dimension. Thus, if a αD element has a limit, this limit corresponds to a lower (α-1)D element.*

The next figures illustrate the concept for 1D elements (Figure 3) and 2D elements (Figure 4). One can see that the 1D elements *limit* corresponds to the two extreme points of this line. However, if the 1D element is closed, than it has no *limit*. The 0D element does not have a limit by definition.

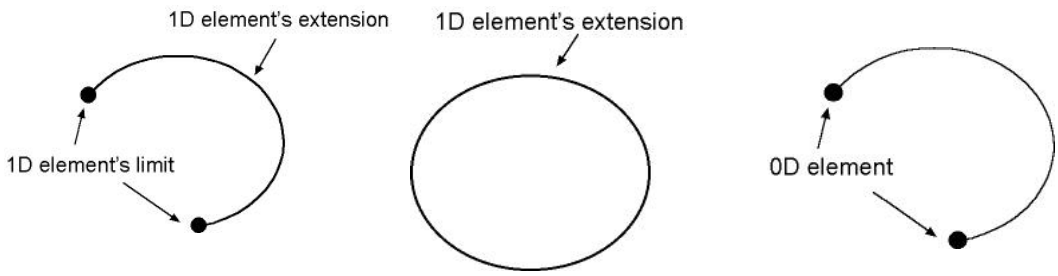


Figure 3: Limit and extension of 1D element.

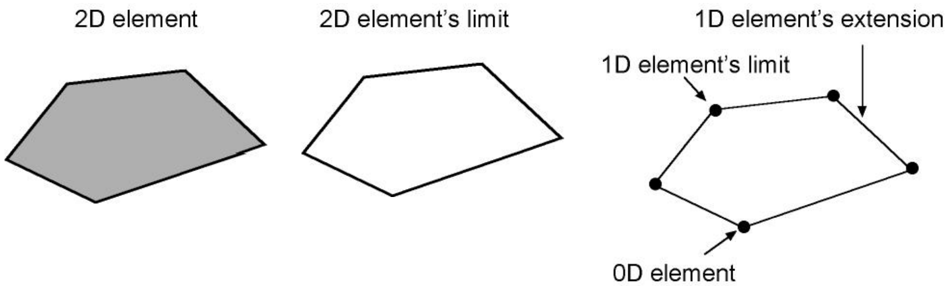


Figure 4: Limit and extension of 2D element.

Furthermore, the 1D element that corresponds to the 2D elements limit (Figure 5) must be understood as a whole and not as five connected 1D elements. The following two definitions refer to dimensional elements with without holes.

Definition 3: A portion of the limit is said to be 1) ***interior limit*** *when it divides the extension and 2)* ***exterior limit*** *when it outlines the extension.*

Definition 4: A dimensional element is said to be ***whole*** *when its exterior limit is continuous. A continuous limit is characterised by a contiguity of all its parts.*

Figure 5 presents a whole dimensional element while Figure 6 presents a non-whole element. These last definitions are especially important for the definitions of spatial object introduced in the following section.

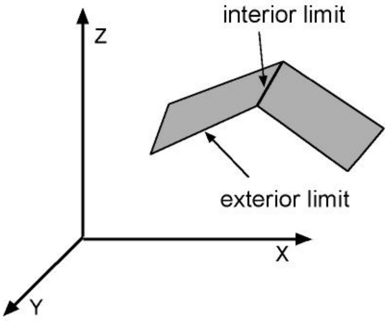


Figure 5: Whole dimensional element.

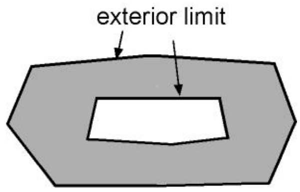


Figure 6: Non-whole dimensional element.

Spatial objects

In this study, for simplicity, we limit ourselves to the third dimension. However, there is no real limitation in the elements dimension. We present the different spatial objects starting from more restrictive notations to less restrictive ones. The definitions presented here are based on the dimensional elements introduced in the previous section.

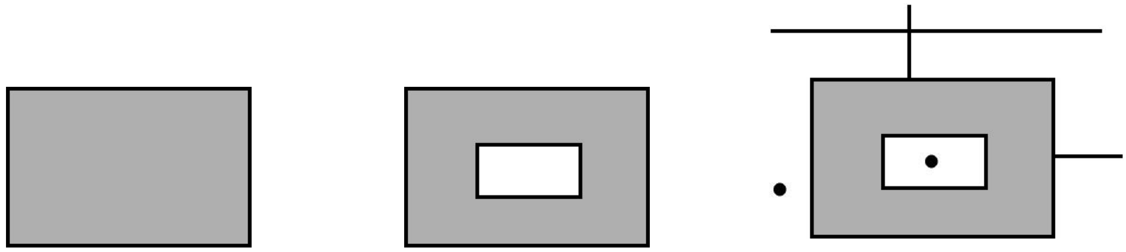
Definition 5: A ***spatial object*** *is composed by at least one dimensional element.*

Definition 6: A ***simple spatial object*** *is composed of continuous series of whole dimensional elements that end with a dimensional element without limit.*

Definition 7: A ***composed spatial object*** *is composed of continuous series of dimensional elements that end with a dimensional element without limit.*

For simple and composed spatial object, each dimensional element is joined to all the other dimensional elements.

Definition 8: A ***complex spatial object*** *is composed of series of dimensional elements that end with a dimensional element without limit.*



Simple spatial object *Composed spatial object* *Complex spatial object*

Figure 7: Examples of spatial objects.

Simple spatial objects are the easiest to understand. The most commonly mentioned in the literature simple objects are: point (composed of 0D element), line (composed of 1D and 0D elements), closed line (composed of 1D element), polygon (composed of 2D, 1D and 0D elements), sphere (composed of 3D and 2D elements) and cube (composed of 3D, 2D, 1D and 0D element). Most of the 2D and 3D data model manages this type of objects.

Point *Line* *Polygon* *Sphere* *Cube*

Figure 8: Examples of simple spatial objects.

The composed spatial object can be seen as the combination of some type of simple spatial objects, e.g. two points, two polygons, etc.

Figure 9: Examples of composed objects.

The complex spatial object can be seen as the combination of simple spatial objects of different kind, e.g. one point and one polygon, etc. It is important to note that even if a spatial object is strictly defined with all the dimensional element (e.g. a cube : 3D, 2D, 1D, 0D element), all of these elements are may be not relevant at a geographic level. Let us consider the real object building that is represented by a simple spatial object cube. In some applications, the 0D element (the cube's corners) may not have a particular interest. Therefore, as we will see in the next section, it is not relevant to take this 0D element into consideration for the determination of dimensional relationships.

Framework for representing spatial relationships

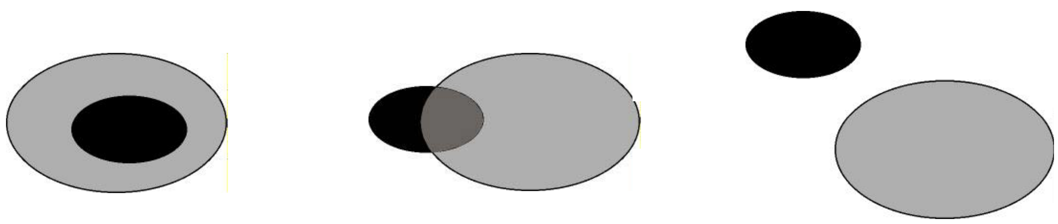
Having the dimensional elements and spatial objects defined in the previous sections, we can now present how this notation can be used to represent spatial relationships. We define dimensional relationships as the relationships that exist between dimensional elements. We define three types of dimensional relationships, i.e. total (t), partial (p) and non-existent (n.e.).

Definition 9: A dimensional element is in ***total relation*** *with another dimensional element if their intersection is equal to the first element.*

Definition 10: A dimensional element is in ***partial relation*** *with another dimensional element if their intersection is not equal to the first element, and if the intersection between the first element and the extension of the second is not null.*

Definition 11: A dimensional element is in ***no relation (non-existent)*** *with another dimensional element if the intersection between the first element and the extension of the second is null.*

These types of relationships are illustrated for two 2D elements (Figure 10).



Total relation *Partial relation* *No relation (non-existent)*

Figure 10: Relations between dimensional elements.

The dimensional relationships between two spatial objects can be applied for all the dimensional elements starting from the one with the highest dimension. For example, the dimensional relationships between two spatial objects polygons (A and B) can be defined in the following order: first, check the dimensional relationship between 2D element of A and all the

dimensional elements of spatial object B; then, check the dimensional relationship between 1D element of A and all the dimensional elements of spatial object B, etc. The following notation is used in the rest of the article:
- R : relationships;
- xD : dimension of the element of the first object;
- y : dimension of the element of the second object.

For example, R2D1 represent the dimensional relationships between the 2D element of the first object and the 1D element of the second object.

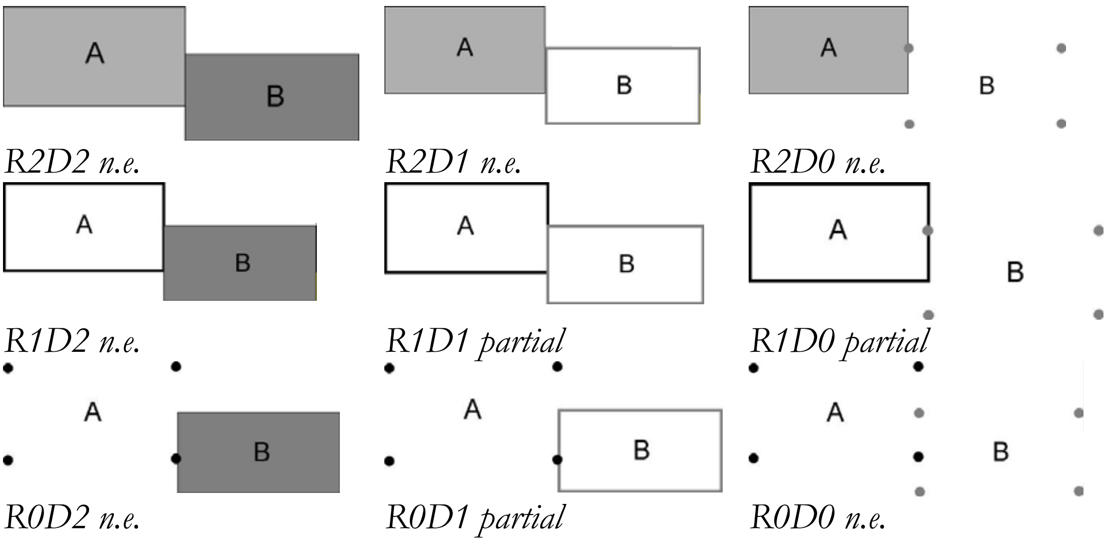


Figure 11: Examples of relationships.

As mentioned before, some dimensional elements cannot be from a geographical point of view. Imagine that one wants to know whether one cadastral parcel touches another one, does not matter if this relationship concerns edges (1D element) or corners (0D element). Then, it is not interesting to look to the four relationships: - R1D1; -R1D0; - R0D1 and R0D0. In this case, it will be better to consider de 1D element without limit and just looking to R1D1 relationships.

There are three groups of dimensional relationships, i.e. *simplified*, *basic* and *extended*. The *simplified* one gives only information about the dimension of the highest dimensional element in the relationships does not matter the type of element of the second object. For example, let us take a cube and interest in its 3D and 2D elements, and a polygon just considering its 2D and 1D elements, we will have one relationship R3D (R3D2+R3D1) and a relationship R2D (R2D2+R2D1) for the cube's elements and a relationship R2D (R2D2+R2D1) for the polygon's elements. In Figure 12, the R3D of the cube is partial, R2D of the cube is partial as the R2D of the polygon.

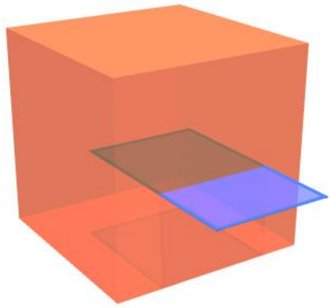
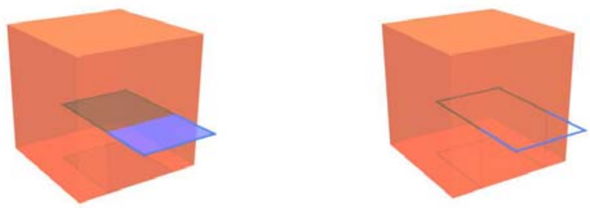


Figure 12: Example of dimensional relationships between a cube and a polygon.

Using simplified dimensional relationships we introduce the *topological relationships* named *inclusion*, *superimposition*, *adjacency* and *contact*. The simplified R3D is called *inclusion*, the R2D *superimposition*, the R1D *adjacency* and the R0D *contact*.

The *basic* relationships can be considered as a further extension of the spatial relationships that give additional information about the dimension of the dimensional elements considered in the relationship. In our example, we would have one relation 3D with 2D (R3D2) (Figure 13a), one relation 3D with 1D (R3D1) (Figure 13b), one relation 2D with 2D (R2D2) (Figure 13a) and one relation 2D with 1D (R2D1) (Figure 13b) for the cubes elements and one relation 2D with 2D (R2D2) (Figure 13b) for the polygons elements. The *extended* relationships take into account the dimension of the intersection (when it exists) between the dimensional objects. In R3, there are 12 simplified relationships, 30 basic relationships and 34 extended relationships.



a. R3D2 and R2D2 *b. R3D1and R2D1*

Figure 13: Basic dimensional relationships between a cube and a polygon.

The presented framework offers an elegant and flexible way of representing a large group of spatial relationships in 3D space. The combination of the freedom in the choice of the geographically relevant dimensional element with the three types of dimensional relationships allows one to decide on working with a very simple set of spatial relationship or on very complex (more than few hundred) ones. The three dimensional relationships can be easily linked to the topological operators based on topological primitives interior, boundary and exterior introduced by Egenhofer (1990). However, the relationships that can be derived applying our framework are far more. Furthermore, we expect the notations and expression used in our framework to facilitate the understanding of a non-specialist for spatial relationships.

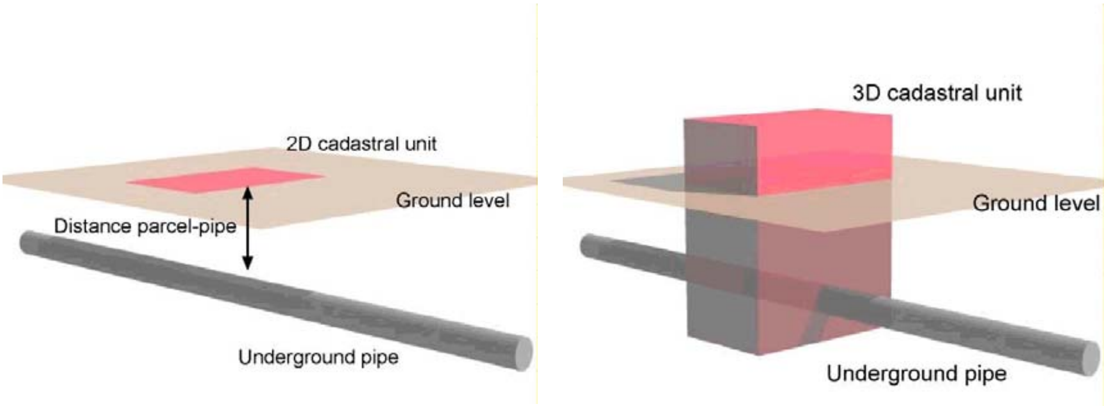
APPLICATION TO 3D CADASTRE

The 3D model is often related to only 3D visualisation. The 3D spatial querying, i.e. one of the key issues of a functional 3D GIS, is frequently underestimated. We present here the potentially of 3D queries and concepts with respect to the 3D cadastral model.

Influence of the object's representation (2D/3D) on the spatial querying

One quite common query in urban planning is to determine all the owners that are affected by geographical phenomena (pipe, road, noise pollution, etc.). The 2D solution is to select all the cadastral parcels that are crossed or touched by the given phenomena. The 2D representation of the phenomena (e.g. road project) is superposed to the cadastral object and the query is "select all the objects that have common parts". Actually, this is a selection of parcels based on a topological criterion. What is the evolution of such a query in a 3D model? One solution to extend the notion in 3D is to keep the same kind of definition, i.e. the 2D cadastral parcel (unit). This is possible because the 3D model do not impose maintenance of only 3D spatial objects, e.g. a polygon (i.e. 2D object) embedded into a 3D frame of reference (i.e. having 3D coordinates) has still 2D dimensionally. Suppose one would like to express the relationships to other objects under or above the ground. Then it would be necessary to either 1) associate to the cadastral unit as an attribute the distance to the other object or 2) to compute the distance between the two objects. Note that the direction of the distance must be clearly defined. In the first case extra attribute information is stored, in the second case, the spatial query requires metric computations (i.e. it is not a topological query). Figure 14 shows an example of a 2D cadastral unit and an underground pipe. The query can be formulated as: "Is the pipe at a particular distance from the 2D cadastral unit".

Another solution is to create a new 3D object, i.e. a 3D cadastral unit that could be represented by a polyhedron. In that case, the query becomes: "Does the pipe intersect (or is included in) the 3D cadastral unit (see Figure 14, 3D)". The spatial query then has the same nature than the initial query in 2D, i.e. it is a topological one.



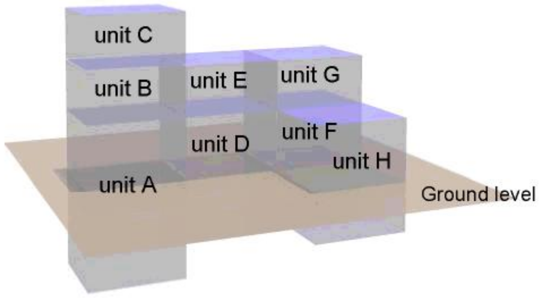
2D *3D*

Figure 14: Query: "Does the pipe go through the cadastral unit".

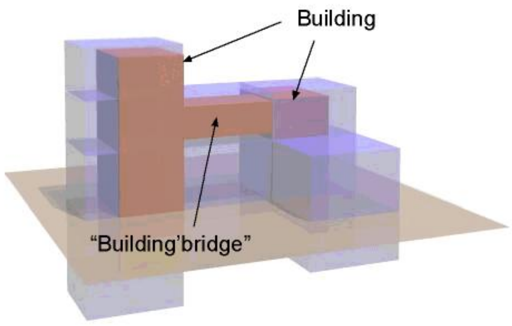
The intuitive conclusion is that 3D spatial queries are of significant importance for the third dimensionality. This is true regardless the choice between 2D cadastral or 3D cadastral unit. But if cadastral unit is defined as 3D object, 3D topological (or more general dimensional) relationships must be supported. Therefore, a 3D topological data structure must be contemplated as an option. Some of our developments toward this direction are presented bellow.

3D Segmentation of space

Using 3D cadastral unit concept may lead to a new approach and a new division of property space. Figure 15 shows a cadastral space partition. We can imagine that the building in Figure 15 is build on (or in !) a cadastral property composed by cadastral unit A, B, C, E, F and G, when the unit D correspond to a road section (property of the municipality). This kind of partition would be also very interesting for technical structure as sewer, pipes, etc. Obviously, that type of approach requires support of 3D spatial relationships and 3D spatial objects.



Cadastral space partition

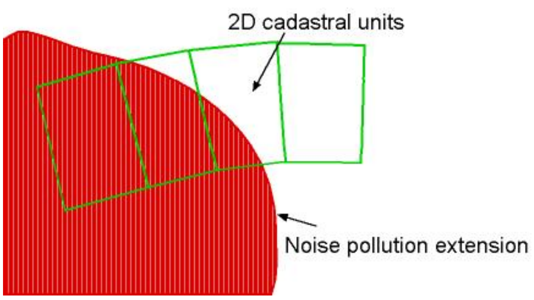


Real building

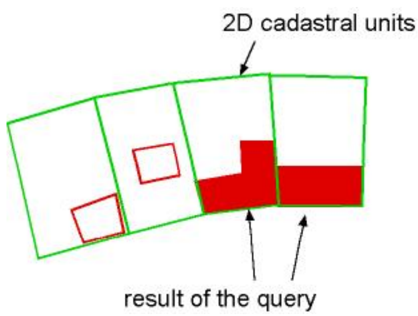
Figure 15: Partition of a building.

The dimensional model and spatial analysis

As presented in section 3.3, the dimensional relationships allow working with a very simple or very complex set of spatial relationships. In the cases of spatial queries mentioned above, these different complexity levels may lead to simplified or very precise description of the relationships between spatial objects. For some queries, the simplified relationships are sufficient. Let us consider the impact of a noise pollution on the cadastral units. If the noise pollution is represented by a 2D spatial object (e.g. a polygon), the only dimensional relationships of interest would be R2D2. Indeed, it is not relevant to refine the spatial relationships at the level of 1D or 0D element. The aim is to know is the cadastral unit is affected by the noise pollution and not to know what are the exact relationships between the borders (edges and corners). In addition, the object "pollution" has rather approximate extension.



"Find the effect of noise pollution"



"Find objects on the street"

Figure 16: Examples of spatial queries in 2D.

However, some complex spatial queries could be necessary when all the dimensional elements play a role, e.g.:

- *Find all the buildings that lay totally on at least one edge of the cadastral unit.* This query will impose to determine if the R2D2 is total, R1D1 partial and R0D0 partial (example given in 2D);
- *Find all the buildings that exceed the cadastral limit and determine the exceeded part.* Expressed by R3D3 partial (example given in 3D). The exceeded part can be retrieve by analysing the relationships between the lower dimensional elements of both objects (2D, 1D and 0D element).

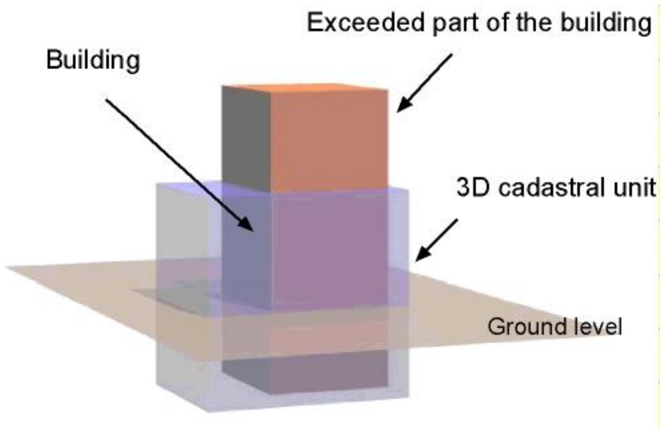


Figure 17: Example of spatial query in 3D.

In both cases, The Dimensional model provides an appropriate answer to the question. See Section 5 for some queries performed in the Oracle database.

Data consistency

One of the major challenges for cadastral administration has always been quality and consistency (incoherence) of data. The consistency of the topological relationships between objects is often considered the major criterion for quality of the model. The examples discussed above referred to the interaction between cadastral objects and other objects (topographic or fictional). But spatial relationships exist also between cadastral objects. For example, an important task of cadastre is to classify the land following a property criterion. By definition, a piece of land may not belong to more than one cadastral unit. This could be controlled by topological relationships between all the cadastral units, i.e. a cadastral unit may not intersect another one. This query can be easily expressed in dimensional terms, i.e. dimensional relationship between higher dimensional elements of two cadastral units (2D or 3D) must be non-existent. In case of a 3D cadastral unit, the relationship will be R3D3 (n.e.) and in case of a 2D cadastral unit R2D2 (n.e.).

It is evident that if 3D cadastral models are envisaged, adapted 3D topology structures (and by extension 3D dimensional structures) have to be adopted. Furthermore, there are different types of incoherence in spatial data. In this research, we are only interested in the topo-semantic ones, which concern topological relationships between objects according to their semantic definition. Figure 18 portrays examples of 2D and 3D topo-semantic incoherence.

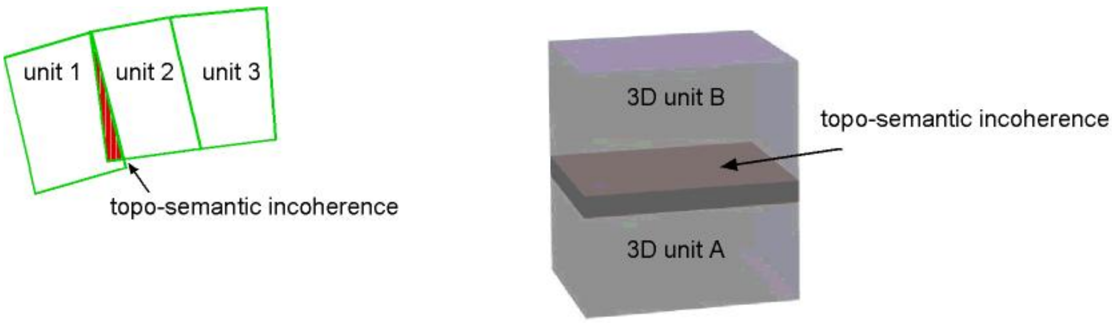


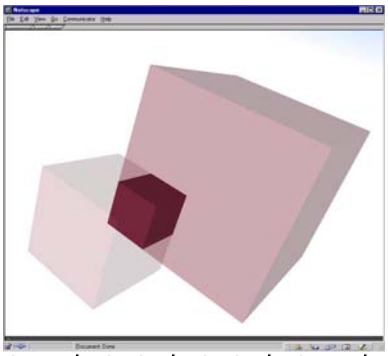
Figure 18: Topo-semantic inconsistency.

IMPLEMENTATION AND TESTS

To be able to test the presented Dimensional model to represent spatial relationships, we have selected a spatial model to represent spatial object. A large number of spatial models are developed and implemented in GIS, CG and CAD systems based on irregular multidimensional cells (*Egenhofer 1989*, *Mäntylä, M. 1988*, *Molenaar 1990*, *Pigot 1995*, *Pilouk 1996*, *Zlatanova 2000*). The names and construction rules of the cells in the different models usually vary. The simplest set of cells is the set of *simplexes*, i.e. *0-simplex (node)*, *1-simplex (arc)*, *2-simplex (triangle)* and *3-simplex (tetrahedron)*. Most of the models allow 1,2,3-cells with an arbitrary shape that imposes some supplementary constraints, e.g. planarity of faces, convex shape (*Molenaar 1990*, *Pigot1995*, *Zlatanova 2000*). The names *node (vertex, point)*, *edge*, *face (polygon)* and *body (solid, polyhedron)* are then used in the literature to denote *0,1,2,3*-cell. Furthermore, the models can be classified into spatial models with explicit representation of objects and spatial models with explicit representation of spatial relationships (topology). Each of the models has advantages and disadvantages that are not to be discussed here. We have concentrated on the Simplified Spatial Model (*SSS, Zlatanova 2000*) basically due to four reasons. First, the model is typical example of explicit representation of objects, which suits our conceptual model. Second, the model has been tested with the framework of Egenhofer for representing spatial relationships that provides a basis for comparison between the two frameworks. Third, the model maintains minimal elements (i.e. node and face) for describing spatial objects and thus simplifies the reconstruction of 3D objects. Fourth, the model has been successfully tested for large 3D models (of size than can be expected for urban models).

We have implemented the concepts of Dimensional model as operators in Oracle database and have tested these operators with two data sets organised according to the construction rules of SSS. The operations are developed with the high level programming language (an extension of SQL) provided by Oracle, i.e. PL/SQL.

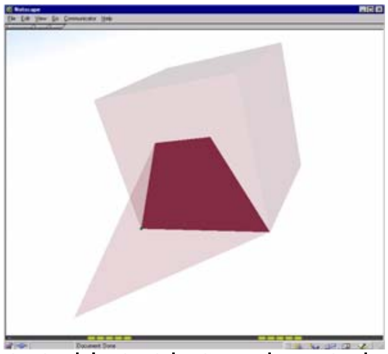
With these dimensional operators, it is actually possible to retrieve basic dimensional relationships between body and body, body and face, body and point, face and face, line and line. The dimensional query returns an answer in a form of relational table that contains all the existing dimensional relationships. Other operators have been developed to extract the precise geometry of the "potential" intersection between spatial objects. Thus we can obtain the precise description of the spatial relationship between two spatial objects and also a correct 3D representation. The 3D visualisation is achieved via java program that translate the Oracle table into VRML file.



R3D3	iR3D3	R3D2	R3D1	R2D3	R2D2	R2D1	R1D3	R1D2	R1D1
3	3	3	3	3	3	3	3	3	3

Figure 19: Example of spatial relationship between two bodies.

The tested queries are: 1) *what is the spatial relationship between two bodies?* (Figure 19) 2*) what is the spatial relationship between a body and a face?* (Figure 20) In the program, numbers decodes the types of relationships, i.e. 1 – total, 2 – non-existent and 3 – partial.



```
R3D2  |R3D1 |R2D2 |iR2D2|R2D1 |R1D2 |R1D1 |iR1D1
3     |3    |3    |3    |3    |3    |3    |3
```

Figure 20: Example of spatial relationship between a body and a face.

CONCLUSION AND FUTURE DEVELOPMENTS

The final objective of 3D GIS is to have a complete 3D model of the reality for topographic and fictional objects. It seems judicious to envisage real 3D concepts for the cadastre by the evolution of the notion of cadastral parcel (2D space's partition) to an extended notion of 3D cadastral unit. Whatever 2D or 3D cadastral objects will be practically selected, a certain level of 3D spatial analysis must be reached. Either topological data structures should be used or 3D spatial operators for non-topological data structures have to be developed. In both cases, the Dimensional model can be used as a framework for the management of 3D spatial relationships. In near future, we will continue experiments with SSS and other data structures toward development of more elaborated 3D spatial queries.

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ABOUT THE AUTHORS

Roland Billen is a PhD candidate at the FNRS (Fonds National de la Recherche Scientifique - National Scientific Research Funds). After his licence in Geography (Survey) at the University of Liege and the Master Degree in Cartography and Remote Sensing (Universities of Liège, Brussels and Louvain-la-neuve, Belgium), he is preparing his doctoral thesis. His research concerns 3D GIS and especially 3D spatial relationships. His basic fields of research are spatial conceptual design and photogrammetry.

Dr. Syka Zlatanova is researcher at the Delft University of Technology, The Netherlands. She has graduated at the University of Architecture, Civil Engineering and Geodesy, Sofia, Bulgaria and has obtained her PhD degree at the Graz University of Technology, Austria. Her interests are in 3D GIS: 3D object reconstruction, 3D data structures, 3D spatial relationships and 3D visualisation. Currently, she is involved in the UbiCom mobile augmented reality project.

CONTACT ADDRESS

Name:	Aspirant FNRS, Roland Billen
Institution:	Department of Geomatics, University of Liège
Office address:	17 allée du 6 Août Liege 4000 Belgium
Telephone:	+32 (0)4 366 57 52
Fax:	+32 (0)4 366 56 93
E-mail:	Roland.Billen@ulg.ac.be

Name:	Dr. Eng. Siyka Zlatanova
Institution:	Department of Geodesy Delft University of Technology
Office address:	Thijsseweg 11 2629 JA Delft The Netherlands
Telephone:	+31 (0)15 278 2714
Fax:	+31 (0)15 278 2745
E-mail:	S.Zlatanova@citg.tudelft.nl