

# Validity of Mixed 2D and 3D Cadastral Parcels

in the Land Administration Domain Model



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Shenzhen - October 2012

# Overview

Introduction

Why Validate?

Definitions and Axioms

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Space Partition

Further Research

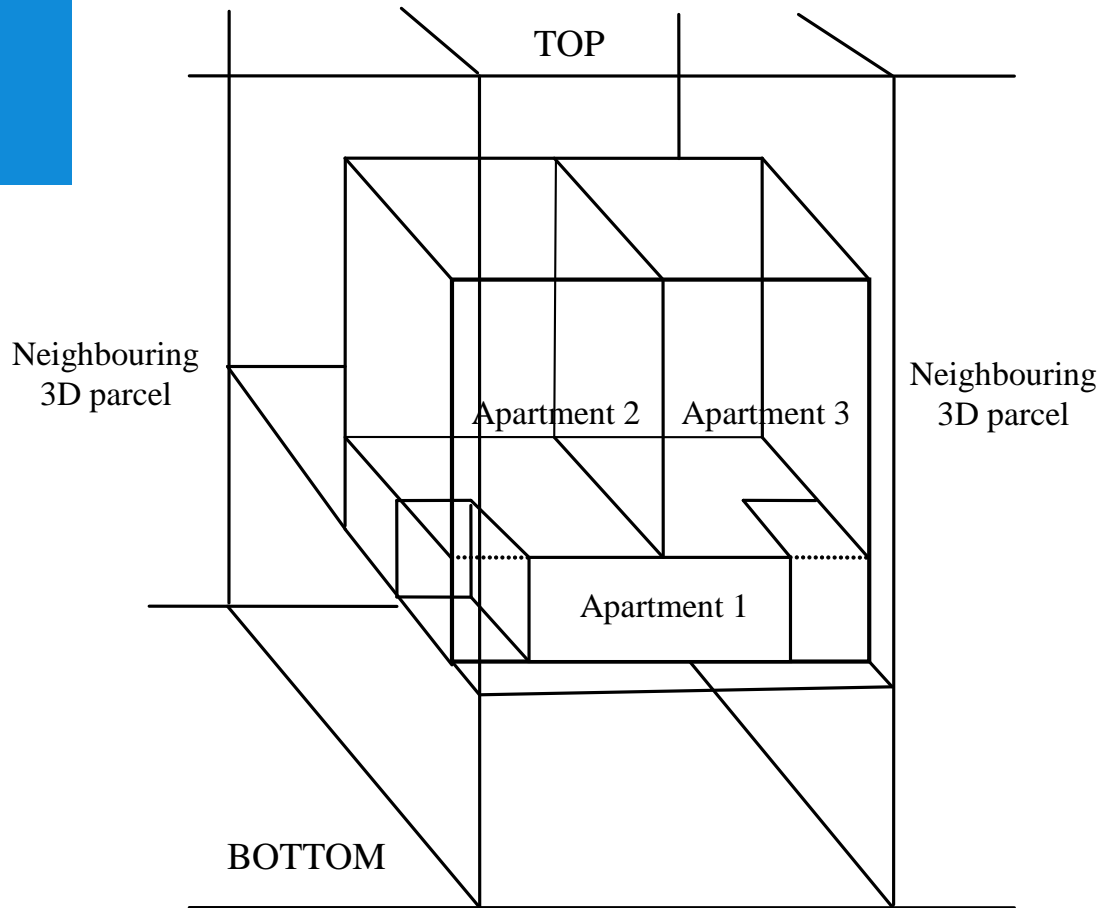
# Introduction

Mixing 3D with 2D Cadastre

The LADM Approach

# Introduction

## Mixing 3D with 2D

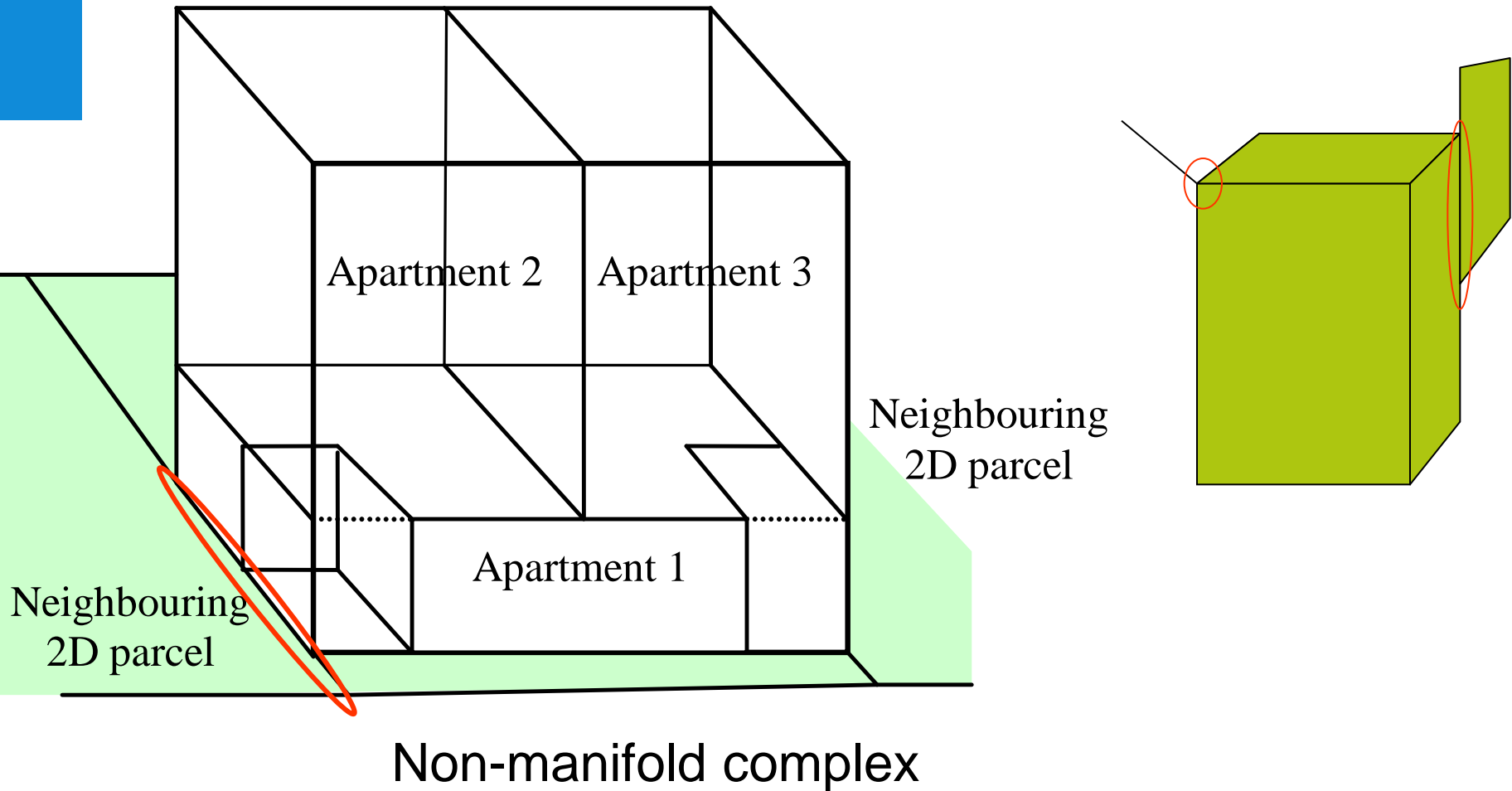


Consider a national park parcel with thousands of points, turned into a 3D parcel.

Full 3D Approach

(Stoter and van Oosterom 2006 Page 41)

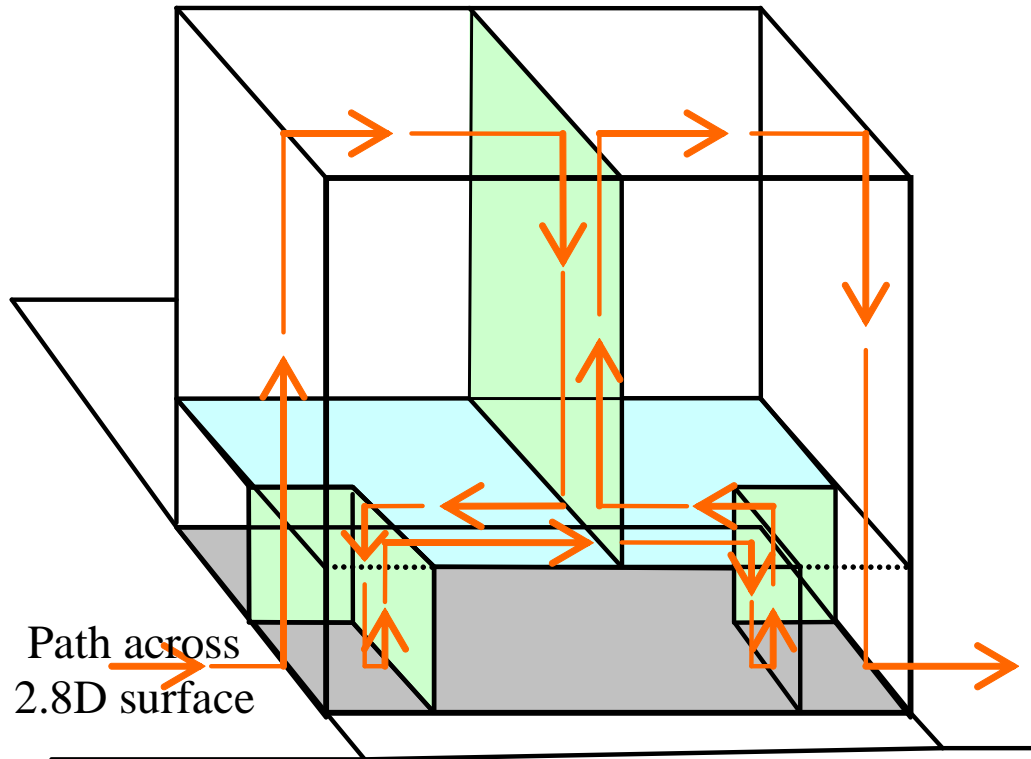
# Mixing 3D with 2D



(De Floriani and Hui 2003)

# Mixing 3D with 2D

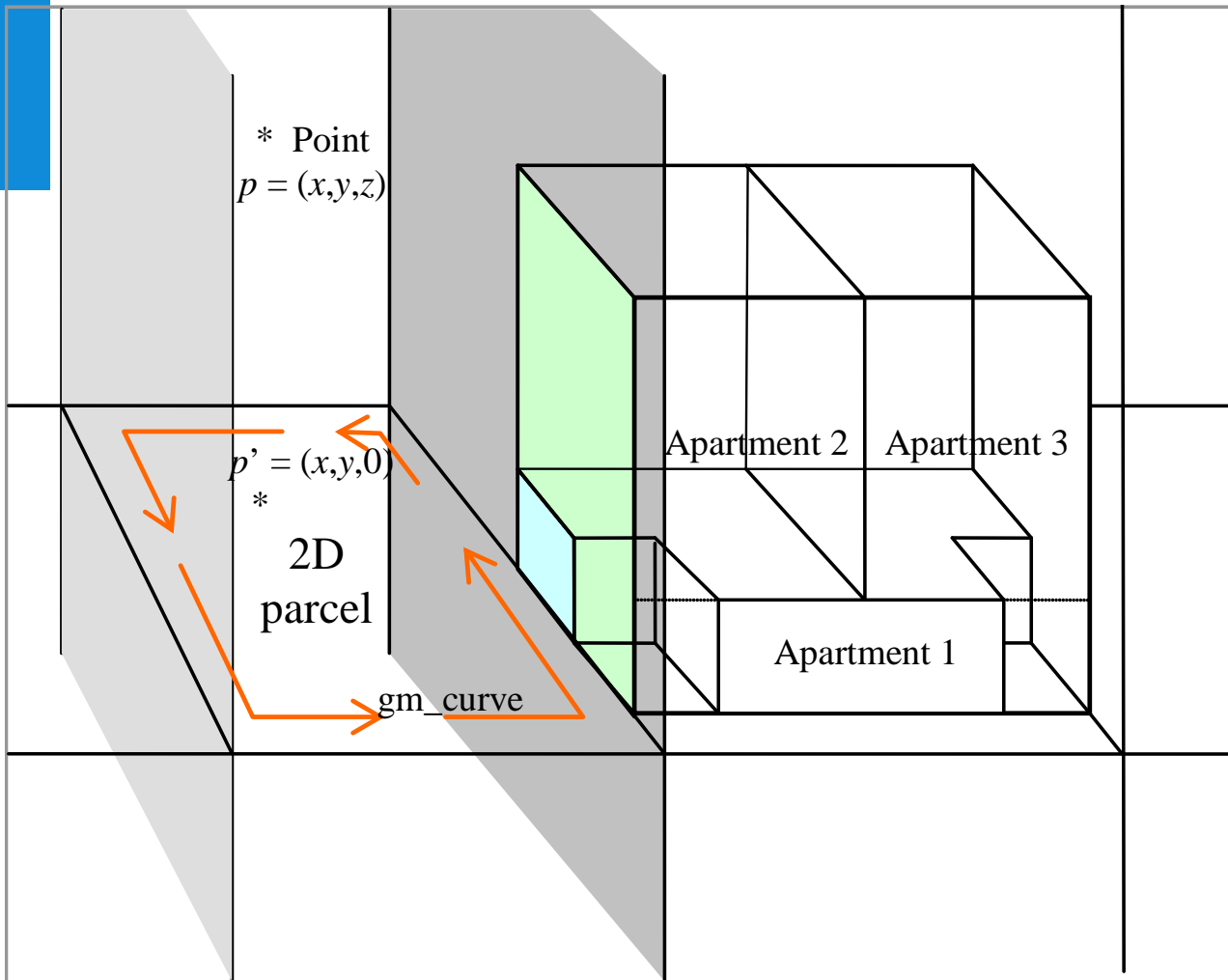
- No floor of the apartment building
- Each common wall stored twice
- 2D stored as 2D
- 3D stored as shells (possibly open)
- Can be validated
- Can't calculate volumes, or determine "inside".



## 2.8D Map

Gröger and Plümer (2005)

# LADM Approach



- 3D Parcels stored as 3D.
- 2D Parcels stored in 2D primitives, but interpreted as 3D parcels.
- Allows unconnected parcels, or topological encoding

ISO 19152

# Why Validate?

To ensure the information is correct?	NO
To ensure our database can accept the data?	Perhaps
To ensure the information is unambiguous?	YES



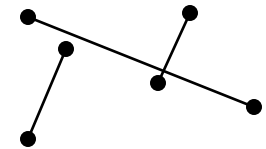
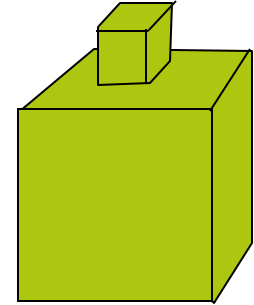
# To ensure the information is correct?

- Sorry, but it is just not possible.
- Information can be valid in all respects, but just plain wrong.

e.g. a perfectly executed plan of a subdivision, but with the wrong parcel identified to be replaced. So it is in the wrong place.

# To ensure that our database can accept the data?

- It can be useful to validate data to allow our databases to accept the data, BUT:
  - Often the validity rules are specific to the vendor.
  - They are rarely (never?) well defined.
  - Sometimes they are unacceptable (especially for an official government specification).
- In any case, we need well defined and meaningful rules.

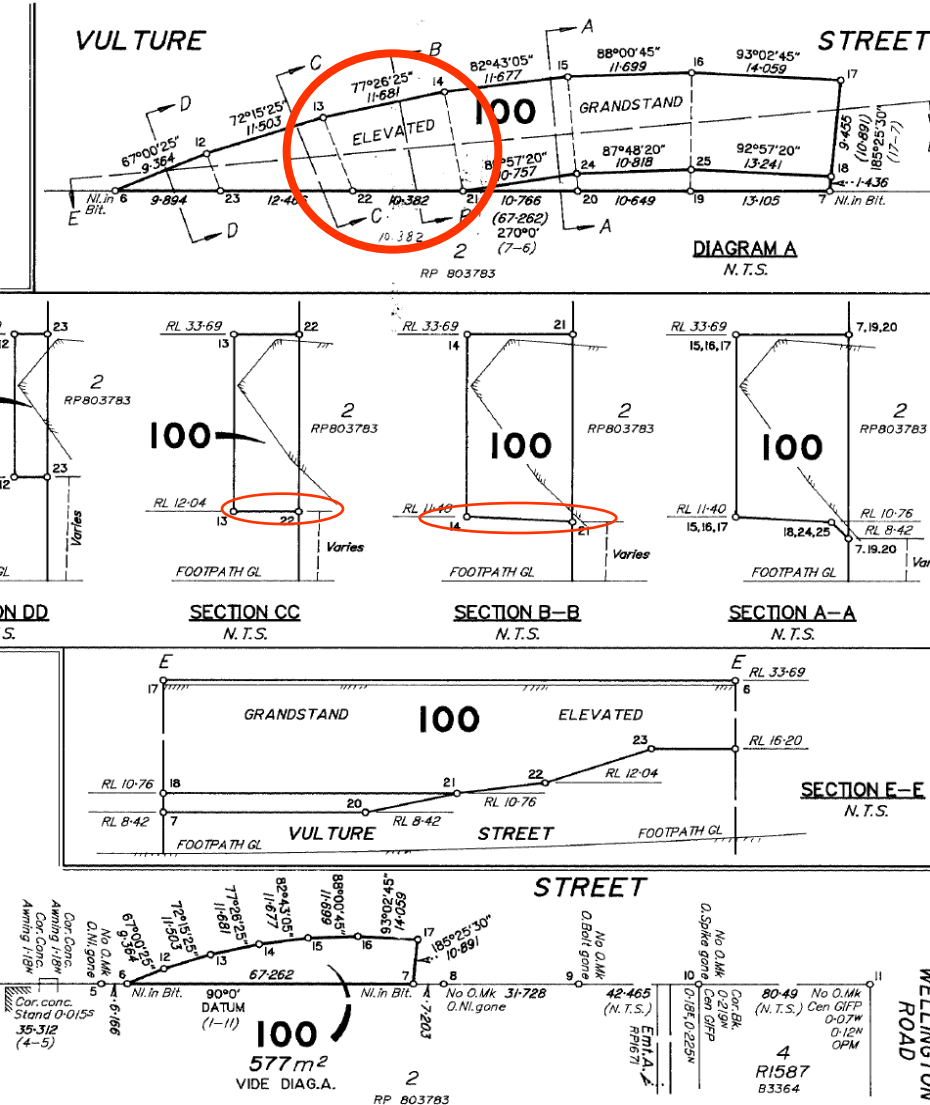


# To ensure that the information is unambiguous

- A cadastral plan is a legal document that defines the extent of a property.
- Any ambiguity about what is included can lead to expensive legal wrangles.

# Ambiguity of Boundary

PLAN MUST BE DRAWN WITHIN BLACK LINES



The lower face highlighted is slightly warped

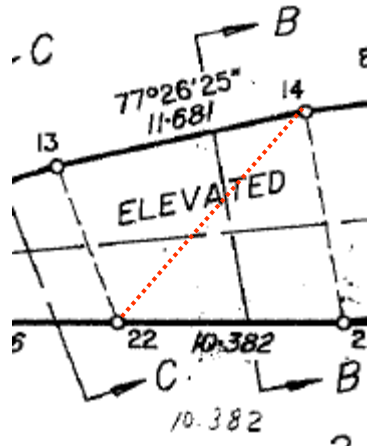
Only by about 30cm

Very hard to see on the plan

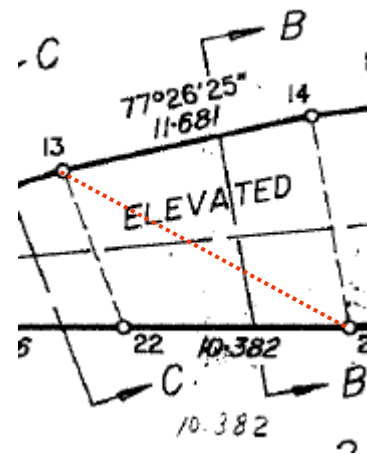
This plan was accepted and is now the legal definition.

The total ambiguity in the plan is at least 15 cubic metres.

# Ambiguity of Boundary



Can fix the problem by triangulating



But if we instead triangulate this way, the parcel gets 10.5 cubic metres bigger

# Definitions and Axioms

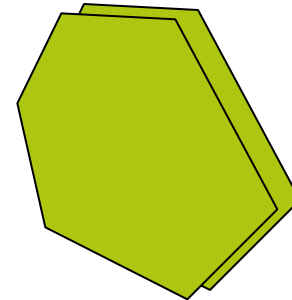
(Brief Restatement)

Completeness, Minimalism, Usefulness

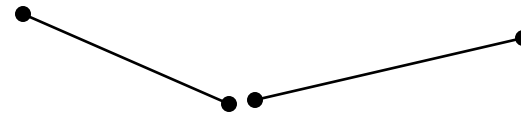
Applicability to LADM

# The Axioms

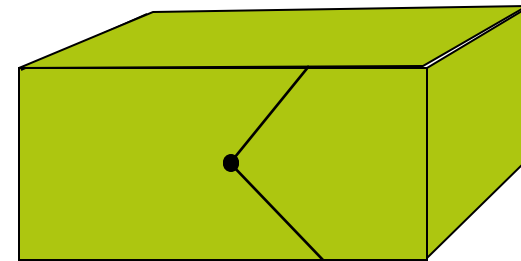
- Axiom A0: For any faces defined on the same set of nodes, the plane parameters must agree.



- Axiom A1: No two nodes are closer than  $\epsilon$  apart.

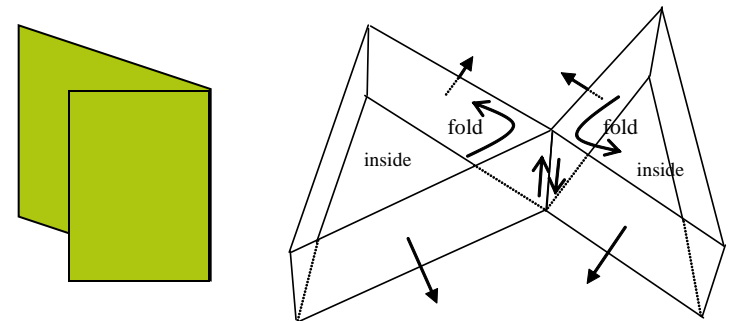
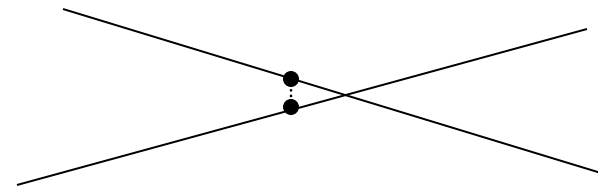
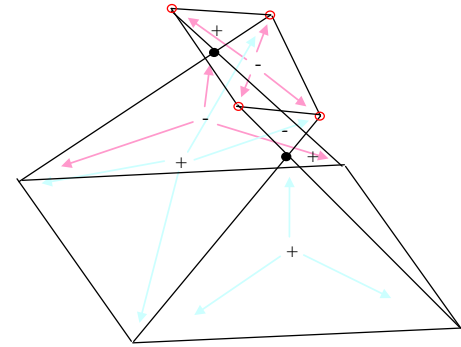


- Axiom A2: Each **finite** node has at least 3 incident faces. (Optional axiom).



# The Axioms

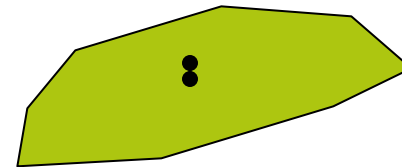
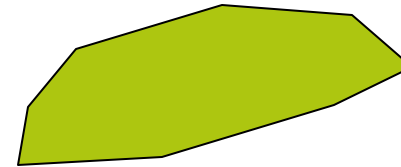
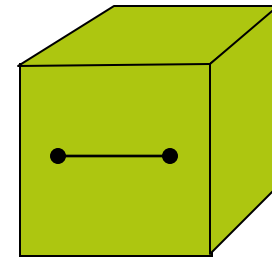
- Axiom A3: The faces incident at a node do not intersect one another except at a common edge.
- Axiom A5: Non-intersecting edges must not be within a distance  $\epsilon$  of each other
- Axiom A6: Every directed-edge of a face in the shell **except those at infinity**, belongs to a fold.





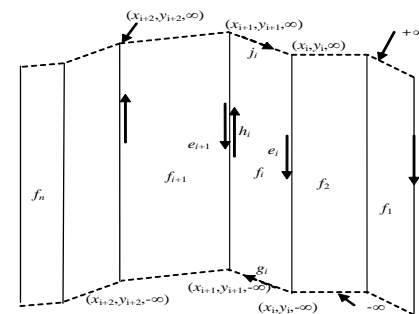
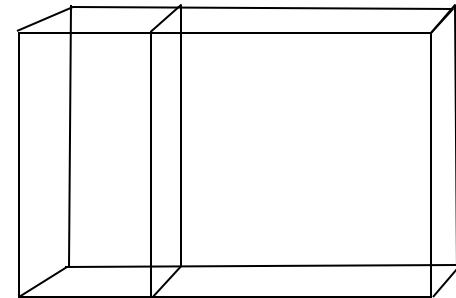
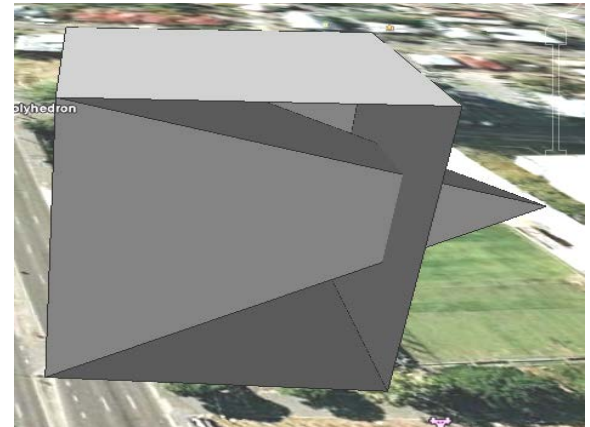
# The Axioms

- Axiom A7: The semi-edges that delineate a hole in a face must be part of the outer boundary of other faces. (Optional axiom)
- Axiom A8: Bounded faces are planar to a tolerance of  $\epsilon'$ .
- Axiom A9: No node is within  $\epsilon$  of a face unless it is part of the definition of that face.



# The Axioms

- Axiom A10: No directed-edge intersects a face except at a node of that edge.
- Axiom S1: No face may be paired with an anti-equal face in the same shell.
- **Axiom AE1: Any open edge must be vertical.**



# The Axioms

- Are they complete?

NO

- Are they minimal?

Perhaps

- Are they useful?

YES

# Completeness of Axioms

- Not really possible
- Further validation rules can always be thought up.
- Also as definitions are refined, new axioms may be needed
  - E.g. A0 – definitions of faces must be consistent.

# Minimal Axioms

- The set of axioms is minimal in that for each axiom, we have provided a test case which fails it, but passes all others.

BUT

- It would be possible to state them in a shorter form (fewer words).
  - We don't, because it is easier to implement the tests as stated here.

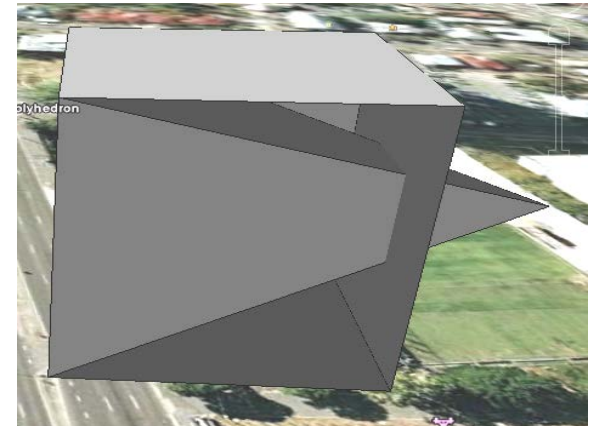
# Usefulness of Axioms

- They provide a rigorous test for ambiguity.
- We believe they can be implemented using restricted precision hardware.
  - i.e. they do not need infinite precision.
- They are built on the assumption of finite precision hardware.
  - i.e. they do not assume that any point can be represented.

# Usefulness

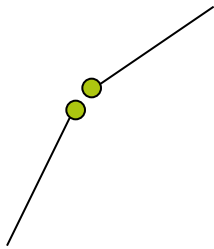
- They are designed for implementation.
- E.g. axioms A1, A3, A5 and A9 could be replaced by a single axiom:

“Axiom AX1 No two faces can cross or approach to within  $\epsilon$ , except at their defined nodes or edges”.

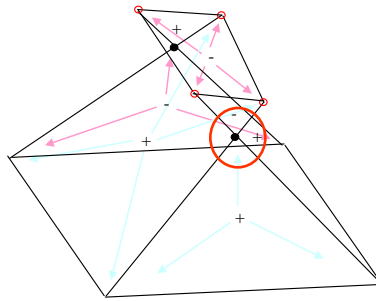


# Usefulness

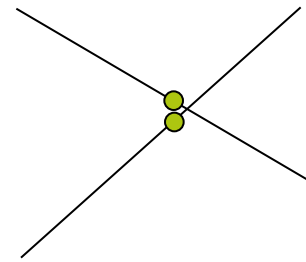
Problem with axiom AX1 is that it is very hard to test. One would need to test every face against every other, and the logic is non-trivial.



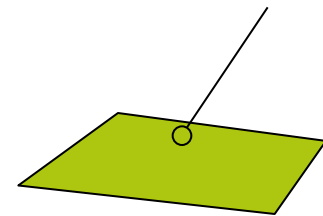
A1 no two nodes too close.  
Many points pairs to test, easy test.  
Can use simple spatial indexing to make quick



A3 Faces incident at a node do not intersect.  
Complex test, but the number of pairs of faces is small



A5 Non-intersecting edges not too close.  
Fairly simple test.  
Can control complexity



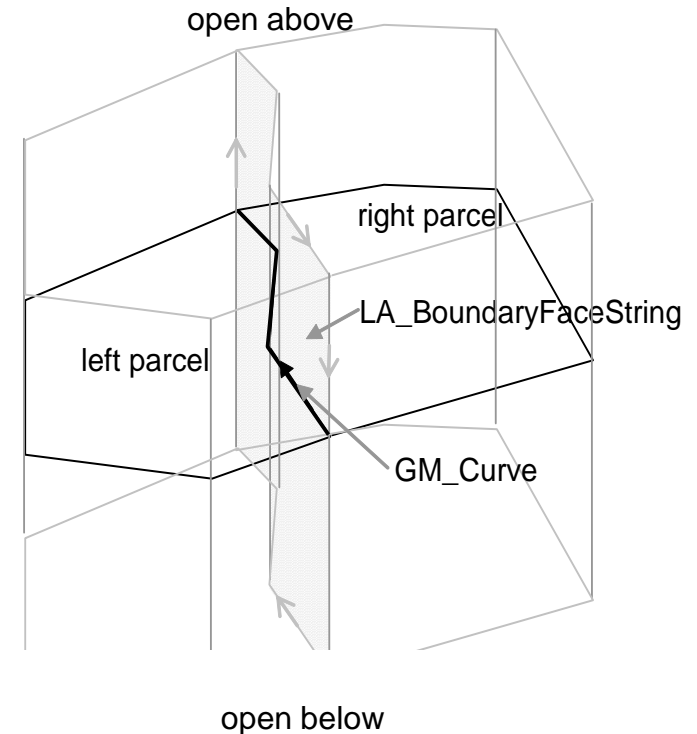
A9 No node too close to a face.  
Simple test  
Can control complexity

No need to test for “edge too close to face”, or “face too close to face”.

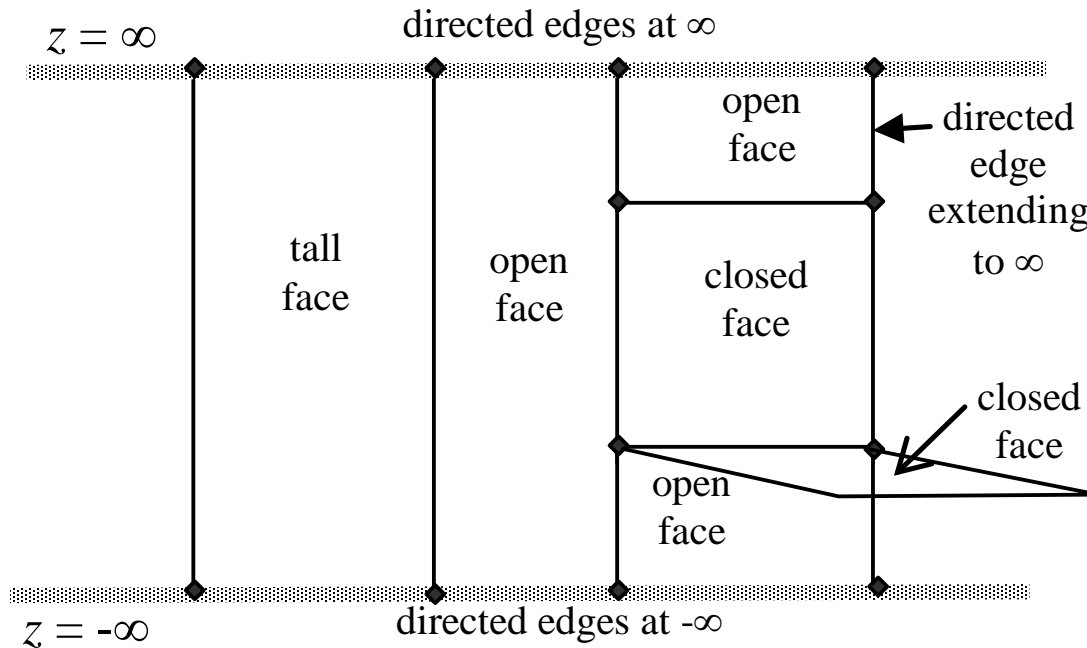


# Axioms for LADM

- Need careful terminology.
- Many parcels in the LADM have no boundary above and/or below.
- So the terms “open” and “closed” are rather overloaded.
- Use the term “cycle shell” to mean a set of faces that define an interior (cf. ISO19107).

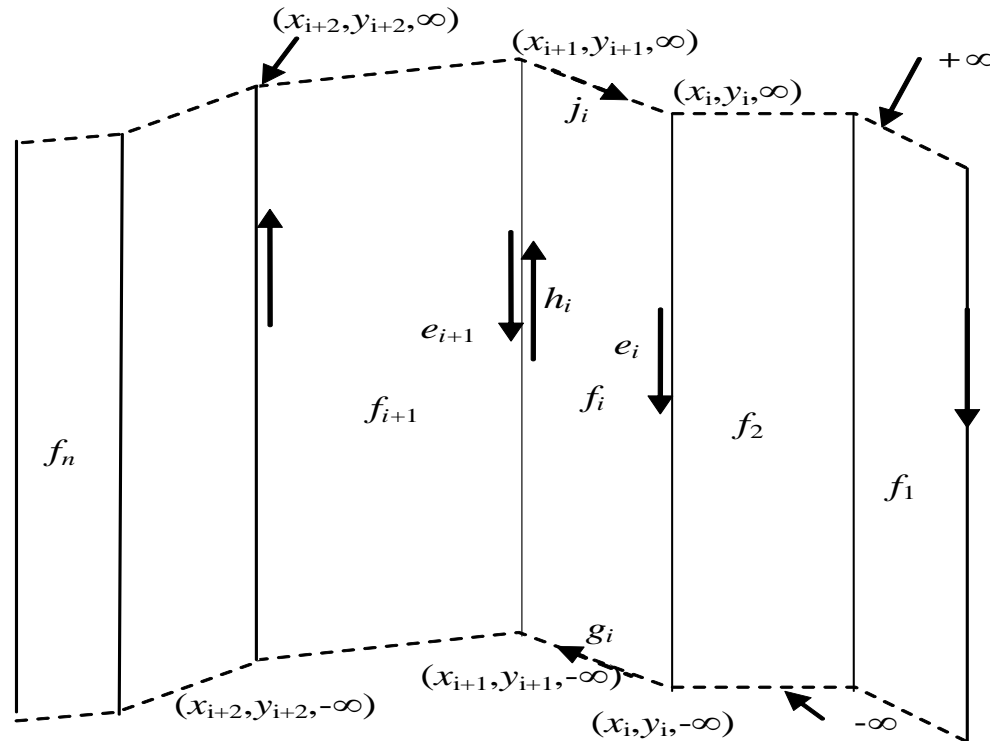


# Types of Faces



- Families of faces which extend to  $\pm\infty$ 
  - Must have vertical edges approaching  $\pm\infty$
- Closed faces do not extend to  $\pm\infty$ 
  - May have any orientation

# LA\_BoundaryFaceString



- Allows a definition of 2D parcels (compatible with 3D parcels)
- Actually stored as a 2D GM\_Curve
- Treated as a set of "tall faces" in axiomatic definitions.

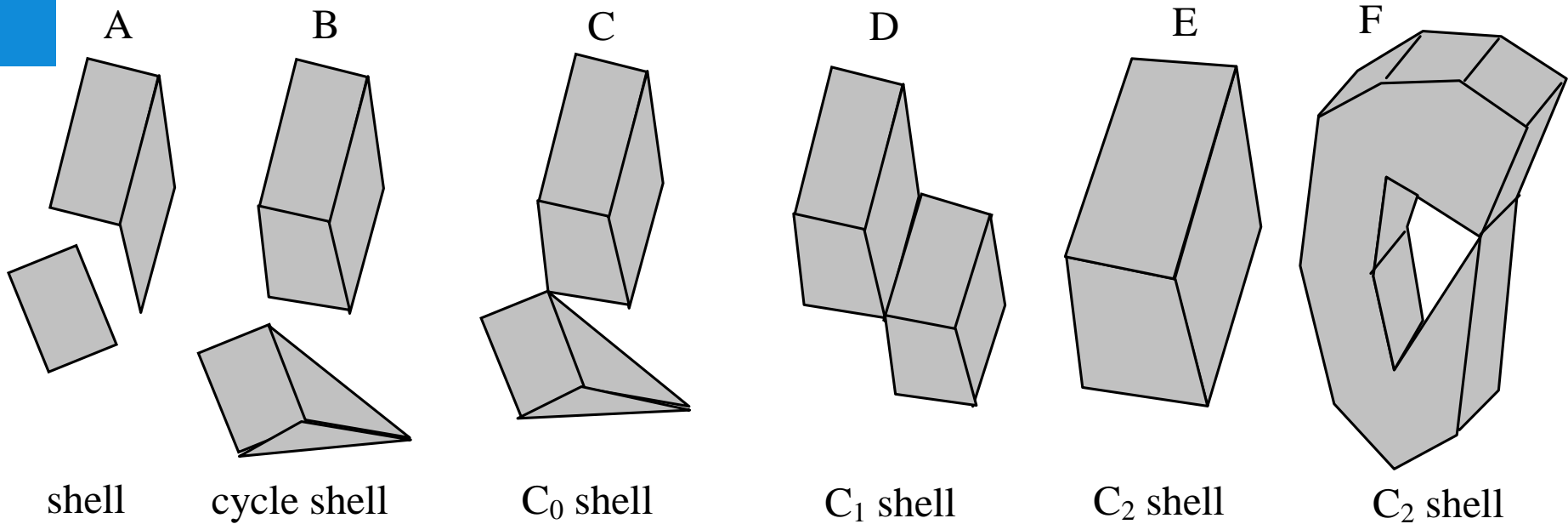
# Connectivity

The axioms will ensure parcels don't overlap

Need to define connectivity

# Types of “Shell”

- A shell is just a collection of faces and their associated edges and nodes.  
Doesn't define a “parcel”



Can define a series of types of shell, becoming more strongly connected. It would be up to the jurisdiction to decide what would be a “valid” parcel. Probably, most would opt for the  $C_2$  (strongly connected) form, breaking the more weakly defined shells into component parts.

# Forming a Space Partition

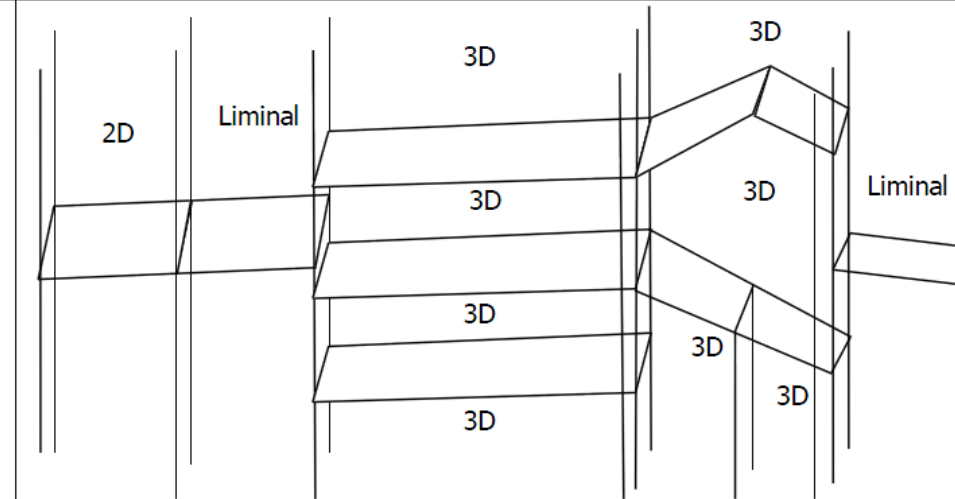
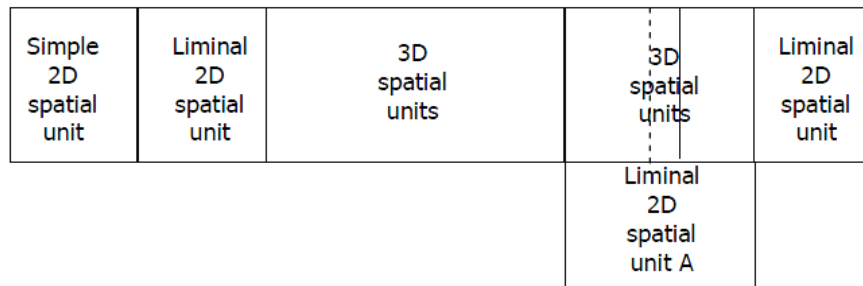
(Ensure every face is paired with an “anti-equal” face)

Liminal Faces

Liminal Parcels

Rest of the World

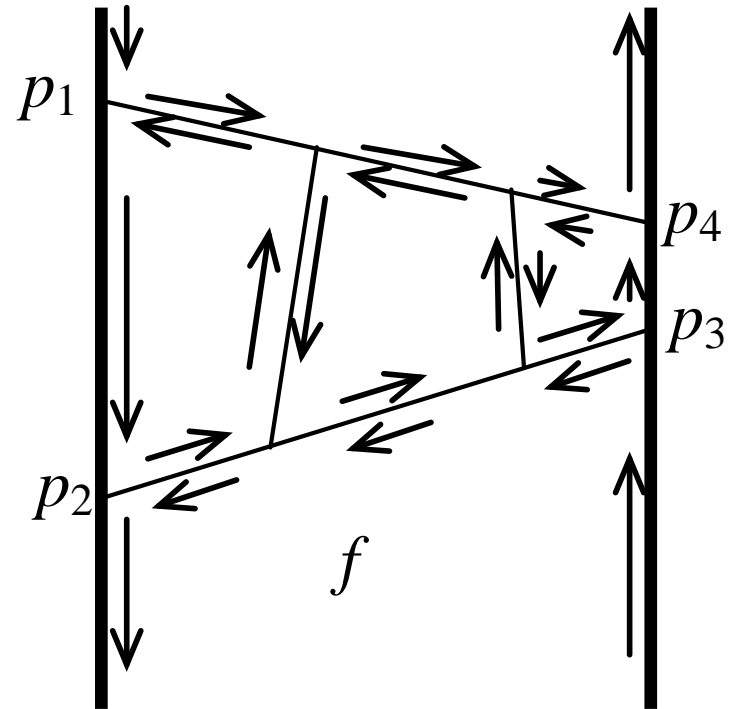
# Liminal Parcels and Faces



- LADM allows for topological encoding of spatial units, defining a possible “liminal parcel” to join 2D parcels with 3D.
  - Allows a 2D parcel to be defined in 3D terms so it can share a face with a 3D neighbour

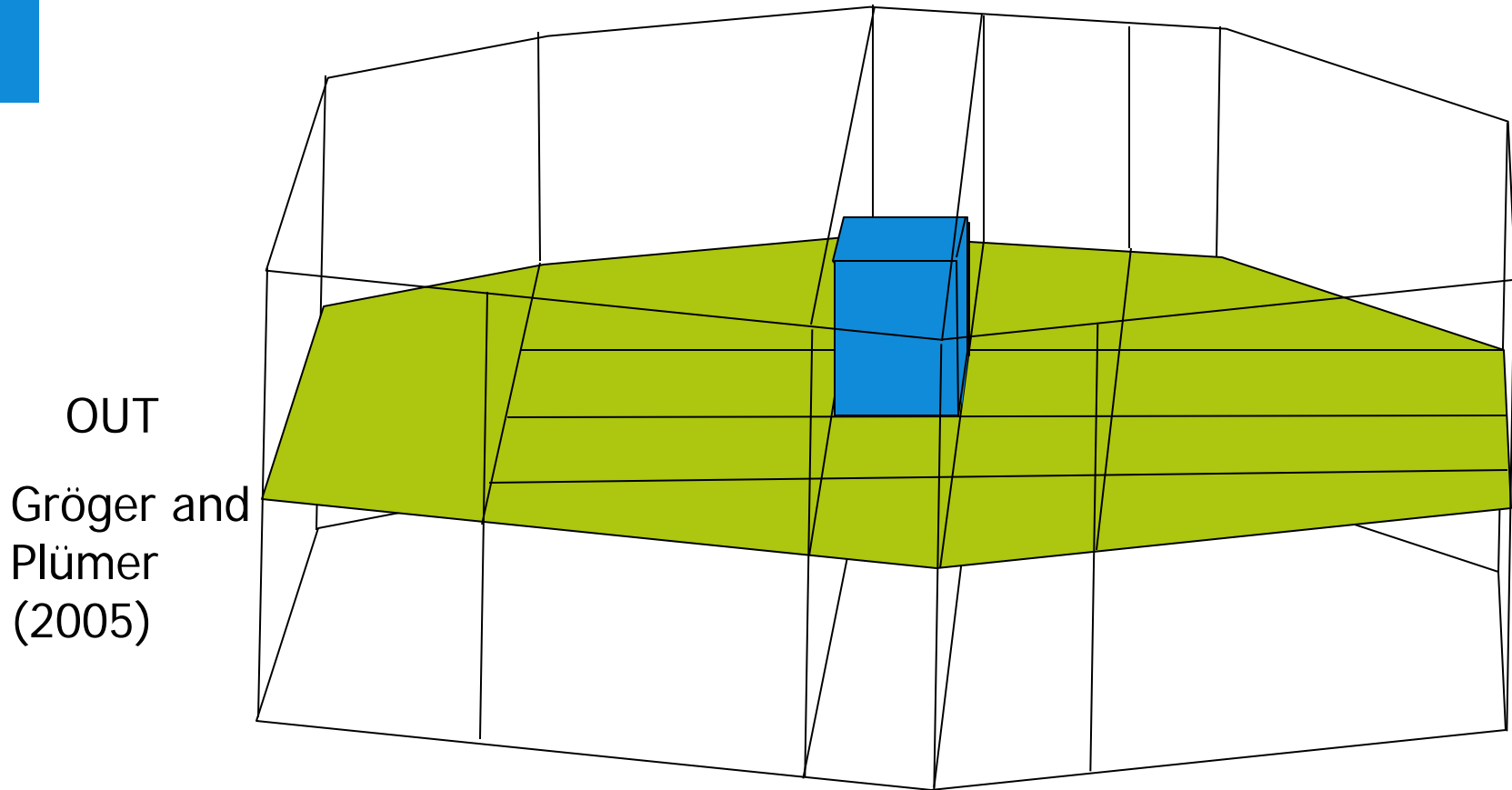
# Liminal Faces

- Fairly simple test for liminal faces
- A liminal parcel is one whose faces are face strings or liminal faces
- This is a definition, and no axiom is needed.



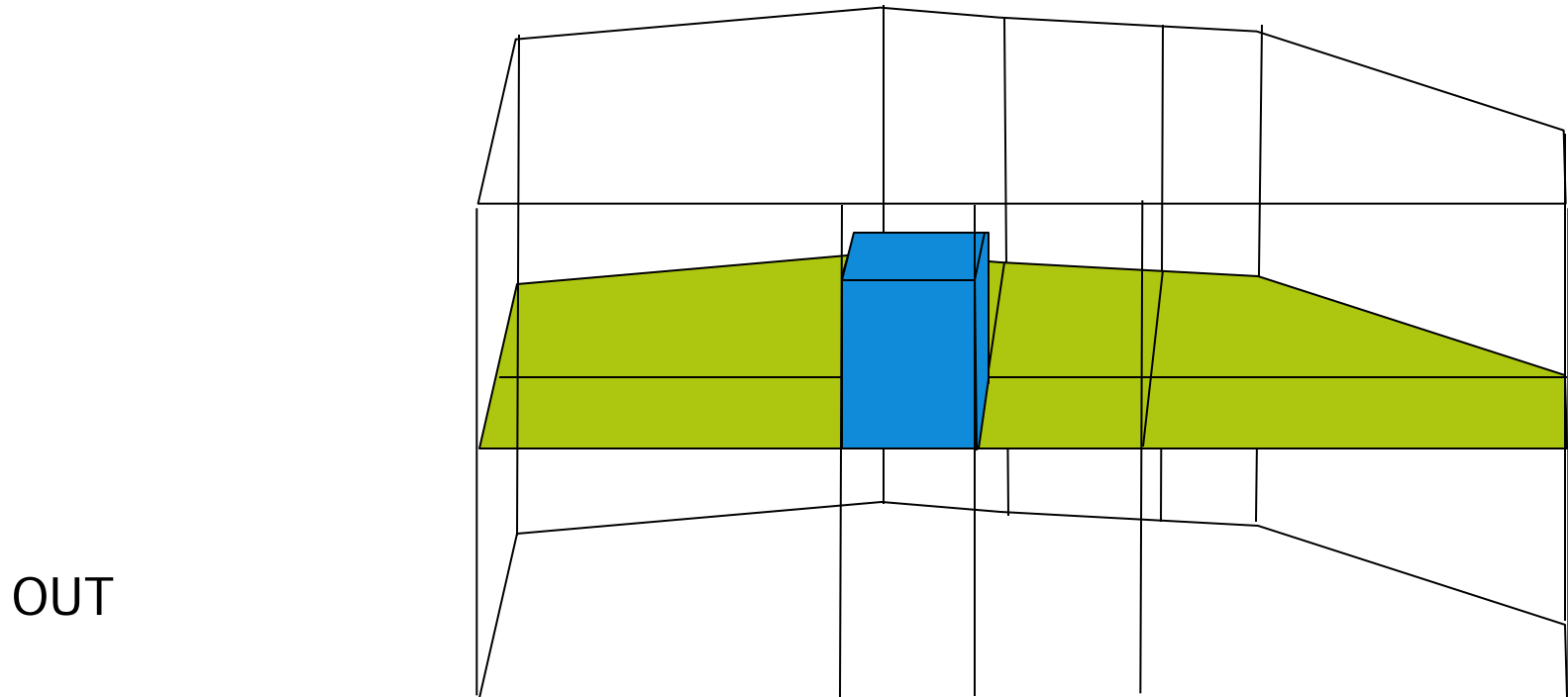


# Rest of the World



OUT is a cycle shell, 2D (?), strongly connected (?)

# Rest of the World



Here OUT is a liminal parcel (because it has faces in its definition that are not “tall”).

# Further Research

- We have not defined a closed algebra.
- Formalise LADM Levels
- Prove proposition that the axioms can be tested reliably using finite precision (floating point) hardware.
- Formalise the definition of “horizontal” and “vertical”.

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