5D Data Modelling: Full Integration of 2D/3D Space, Time and Scale Dimensions

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Abstract. This paper proposes an approach for data modelling in five dimensions. Apart from three dimensions for geometrical representation and a fourth dimension for time, we identify scale as fifth dimensional characteristic. Considering scale as an extra dimension of geographic information, fully integrated with the other dimensions, is new. Through a formal definition of geographic data in a conceptual 5D continuum, the data can be handled by one integrated approach assuring consistency across scale and time dimensions. Because the approach is new and challenging, we choose to step-wise studying several combinations of the five dimensions, ultimately resulting in the optimal 5D model. We also propose to apply mathematical theories on multidimensional modelling to well established principles of multidimensional modelling in the geo-information domain. The result is a conceptual full partition of the 3Dspace+time+scale space (i.e. no overlaps, no gaps) realised in a 5D data model implemented in a Database Management System.

Keywords: Multidimensional data modelling, spatial DBMSs, spatial data types, spatio-temporal data models, multi-scale data models, 3D data models.

1 Introduction

The role of geographic information in our society has changed tremendously the past decades. From being collected and used ad hoc for maps and other specific applications, it has now become part of the Geo Information Infrastructure (GII). Ultimately GIIs will serve complete information flows in the web from observing/monitoring via processing/analyzing/planning to communicating and controlling actions. After many isolated initiatives, our society is now heading towards a sustainable GII in which geo-data is shared and re-used by many and highly different users and applications through machine-based linking of large amounts of distributed geographic data and information.

Formal definitions of geo-information are required to enable understanding and satisfying the requests of people and (increasingly) machines to use appropriate geo-information available within the web. This paper focuses on formalising the data models and structures that capture geo-information. Data structures for geo-information bring specific challenges as traditional DataBase Management System (DBMS) implementations for non-dimensional information are not capable of

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handling the different dimensions of geo-data sufficiently. In our approach we distinguish five dimensional concepts of geo-data.

Apart from their location and 0D to 3D geometrical and topological characteristics, geo-data has further temporal (when was a moving object at a specific location; when was an object valid in the database?) and scale components that were often implicitly taken into account when the data was collected. These different dimensional aspects highly correspond, e.g. a (possibly geometric) change may be only relevant for the highest scale of an object or understanding the route directions for a long car trip requires overview, but at specific locations (e.g. to rest or to stay overnight) consistent information at a higher scale, with also temporal information (i.e. weather conditions at a certain moment) may be needed. Although (multi-)scale is a well-known concept in the geo-information technology domain, regarding it as an extra dimension of geo-data, integrated with the other dimensions, is new.

Despite the interdependencies, until now different dimensions of geo-data have been studied in separate initiatives, with sometimes limited support for the other dimensions. Although these past studies have gained important knowledge on how to handle the individual dimensions 2D/3D space, time and scale, no modelling approach exists that truly integrates all dimensional concepts of geo-data at a fundamental level.

This is our motivation to start a new research on a conceptual full partition of 3Dspace+time+scale (i.e. no overlaps, no gaps) realised in a true 5D generic model. This paper elaborates on the research methodology that we propose to accomplish the true 5D model. In contrast to a separate handling of spatial, temporal and scale dimensions, a true 5D approach provides a sustainable and solid foundation for the GII for three main reasons:

- The deep integration of all dimensional concepts accomplishes a highly formal definition of geo-data (with 5D data types and 5D topological primitives) as the relationships between space, time and scale aspects of geo-data are fully addressed and no special cases need to be treated in another way anymore. Every case will be a specialisation of the model.
- The model enforces consistency crossing dimensional borders which improves the quality of geo-data.
- Optimal efficient 5D searching and maintenance can only be realised if a 5D data type and index/clustering is used, otherwise the DBMS query plan has to select first on space, then on time and then on scale (or in another order). An example of a 5D search that appears in space, time, and scale context is the integration of a database with physical plans at different moments in history and a database with historical information on buildings to check which buildings (extensions) conflict with which status of the physical plan. Another example of a 5D search is comparing the cadastral database that registers the legal status of networks based on the physical extent of the network at the moment of registration with the physical registration maintained by the network company in which changes as extensions, deletion and movements of parts of the network are recorded as well.

In our approach the multidimensional integration is studied at two levels. At first a conceptual 5D data model will be designed on which all other geo-data models can be founded. Secondly, as the foundation of the GII consists of geo-DBMSs maintaining geographic information, the methodology proposes to study how DBMS functionality can be extended up to 5D as implementation of the conceptual model.

We do recognise the high ambitions to realise a true 5D data model. However our aim is to lay down a foundation for multidimensional data modelling by defining a theory validated through prototype implementations, which can be further developed in the future. In addition, in a step-wise approach we will apply mathematical theories on multidimensional modelling to established principles in 2D/3D, time and multi-scale modelling. By studying several combinations of the different dimensions, we will accomplish the optimal method for including multidimensional concepts and notions in geo-data modelling.

To explain our proposed methodology in which we combine multidimensional principles established in both the geo-domain and mathematical theories, we first elaborate on the multidimensional modelling concepts and principles that are studied in the geo-information domain (section 2), while in section 3 we will explain the potential mathematical theories on multidimensional modelling that we will explore. Section 4 explains our proposed methodology in more detail and the paper ends with discussion in section 5.

2 Principles of Multidimensional Modelling of Geo-information

This section firstly describes in Section 2.1 how earlier work studied the various dimensional aspects of geo-information. Sections 2.2 to 2.4 describe the principles established in 2D/3D, spatio-temporal and multi-scale modelling separately, which form the point of departure for the true 5D data model in our research.

2.1 Dimensional Aspects of Geo-information in Previous Work

The high correspondence between 2D/3D spatial, time and scale characteristics of geo-information has been recognised by other researchers. A well-known example is [1] who studied the question ‘How long is the coast of Britain’. Based on empirical evidence he found out that the measured length of a coastline depends on the scale of measurement: the smaller the increment of measurement, the longer the measured length becomes. This is because smaller increments allow a more curvilinear route. The coastline example can be realistically extended into 5D by including 3D space and time (besides the scale): what is the volume of above sea-level a 100m wide ‘strip’(inland) and how is this evolving over the last 50 years?

Other researches that studied the varying perception and meaning of concepts dependent on the scale of observation are [2, 3, 4, 5, 6, 7]. We propose a data model and structure that makes these dependencies explicit.. This will make it possible to fully support 5D applications, for example efficient and consistent monitoring of coastline change at different scales.

Apart from initiatives that focused on the scale dimension of geo-information, several previous researches have studied multidimensional modelling of geo-information.

A first related topic is nD storage and mining [8, 9] which aims at aggregating information on multiple thematic (non-spatial) attributes to perform efficient database querying subsequently. For example 5D data would be the result of combining object-id, weight, colour, price, and date attributes. Although this 5D data focuses on thematic attributes in data mining and not on dimensional concepts in geo-data modelling, the similarity is that it also considers multiple aspects of data in an integrated manner.

Also the domain of nD modelling is related [10, 11]. nD modelling extends BIMs (Building Information Models) with additional thematic characteristics to serve each stage of the lifecycle of a building facility through one common information model. However nD modelling focuses mainly on integrating multiple thematic, and not multidimensional, concepts. As the multidimensional data model that we propose offers a framework to structure any geo-information, the thematic approach of nD modelling can be well served by progresses in multidimensional data modelling, for example 4D BIMs that include the time dimension.

2.2 2D/3D Modelling

In 2D/3D spatial modelling we can observe the following developments.

The Open Geospatial Consortium (OGC) and ISO establish standards for handling spatial data, which has resulted in Simple Features Specifications for Structured Query Language (SQL) in 2003 [12, 13, 14]. These specifications define how to support 0D, 1D and 2D spatial objects (that can be defined in 2D and 3D space) in object-relational DBMS environments. The specifications also define operations to detect eight topological relationships defined in the 9-intersection framework of [15], i.e. equals, disjoint, intersects, touches, crosses, within, contains and overlaps. Mainstream DBMSs have implemented these specifications, resulting in a shift from ad hoc use of geo-data to interoperable geo-data as part of generic information flows.

Currently no standards exist for implementing 3D geometry and topology in DBMSs. However 3D data structures achieved in research have shown that geometry and topology of 3D objects can be structured in several ways: polyhedral [16, 17, 18] regular polytopes [19] and TEN (Tetrahedral Network) [20]. More research is required to see what applications can be served best by what kind of 3D data structure, how to define standards for more advanced 3D geometry, 3D topological primitives and 3D topological relationships and how to enforce the validity of such data, see also [21, 22, 23, 24].

Many initiatives have studied modelling the 3D concepts of geo-data in information models driven by applications such as facility management, urban planning, 3D cadastre, noise modelling, flooding, disaster and crisis management. To unify these initiatives, OGC and ISO TC 211 established a standard for exchanging 3D city information in 2008 [25, 26, 27, 28]. This information model, called CityGML and based on the Geography Markup Language (GML), provides a common definition and understanding of basic entities, attributes and relationships of 3D city objects. GML provides classes for 0D to 3D geometric primitives, 1D-3D composite geometries (e.g. *CompositeSurface*), and 0D-3D geometry aggregates (e.g. *MultiSurface* or *MultiSolid*). The geometry in CityGML follows the ISO 19107. Generally volumetric objects are possible, but the validity of closed volumes cannot be enforced in CityGML. In addition 3D topological structures are not standardised in CityGML. Currently CityGML only defines relationships between face-sharing features via a reference to a common outer shell polygon and between feature classes and the terrain through the terrain intersection curve [28]. Also the vector (and specifically simple feature) representations may not be suitable to represent every type of 3D object [29].

The time and scale dimensions are handled in CityGML, however not in an integrated manner. The time dimension is separately handled by adding attributes to geometrical objects, i.e. *creationDate* and *terminationDate*. Scale is handled via the Level-of-Detail (LoD) concept. CityGML models multiple geometrical representations of the same real world with increasing accuracy and structural complexity (LoD0 to LoD4). However the different LoDs are poorly connected and therefore consistency between different LoDs cannot be assured. In addition different LoDs are maintained in different representations, i.e. on the fly derivation of lower LoDs from a higher LoD is not supported and higher LoDs cannot consist of parts from a lower LoD [30]. A final shortcoming of LoDs in CityGML is that the indoor-level (LoD4) is not well represented. For example what is the inside of building: a hole (inner polygon) in the building object or another world that should have its own LoDs [30, 31]?

2.3 Spatio-temporal Modelling

Temporal aspects of geo-data is fundamental for recording or monitoring changes, for describing processes, and for documenting future plans. For example monitoring the status change of a set of related features (Figure 1, left) or monitoring changes of moving objects (Figure 1, right).

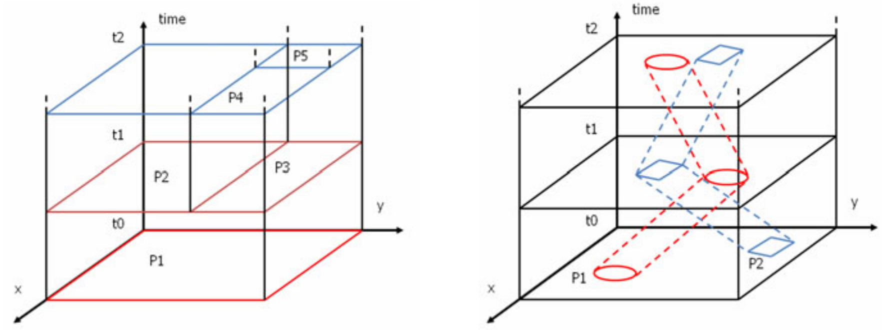

Fig. 1. Time as third dimension: division of parcels (left) and moving objects (right) [32]

Many Spatio-Temporal (ST) data models have been designed to model changes of geo-information [33, 34, 35]. The semantics of the time dimension included in these models vary from model to model and generally address the following items:

Temporal granularity specifies to which units of data one temporal attribute is added, e.g. whole dataset, object class, object instance or attribute.

Temporal operations for spatio-temporal analyses.

Modelling foundation for time describes which type of changes can occur to the value of a thematic or geographic characteristic, i.e. discrete changes or more continuous/gradual change.

System (or transaction) time indicates the time an event is recorded in the database.

Valid (or real-world) time describes the time that an event happened in the real world.

Lifespan identifies the history track of real world objects. Some events last only one short moment, e.g. an explosion or a traffic accident, which are like point objects. Other situations last for a longer period of time, e.g. the fact that a building has a particular owner, which are like linear objects representing a time interval.

Representation of time can differ from maintaining the duration of the status of an object (i.e. period) to recording events (i.e. start- and end-moment) that imply status change.

For ten well-known ST models, Table 1 shows how they represent the time dimension [36].

Table 1. Representation of time in ten well-known ST models, after [36]

Spatio-Temporal Data Models	***Representation of Time***	
	Models	***Time as***
Snapshot model	Layers- Snapshots	Attribute of location
Space-Time Composite	Polygon history	Integral part of spatial entities
Simple Time stamping	Object's Creation – Cessation	Attribute of the object
Event Oriented	Events, change	Attribute of an event
Semantics, space and time separately in 3 Domain	Temporal versions	Independent object
History Graph	Events, processes	Attribute of objects, events
ST Entity Relationship (STER)	Entity change	Attribute of entity, relationship
Object Relationship Model	ST phenomena	Attribute of Object
ST Object-Oriented	Object Change	Attribute of object
ST UML	TimeUnit	Via the Specification box
Moving Object Data Models	Functions	Integral part of spatial entities

The deep integration of time with space and scale concepts will fully handle changes upon position, attributes and/or extent of the objects in the unified space-time-scale continuum. Some aspects require specific attention for the time dimension in this deep integration. At first all possible changes of geo-information at varying scales should be well represented, i.e. change in geometry OR topology OR attributes, or no change at all. In addition the integrated representation of time should not only support changes at discrete moments, as currently supported by most of the ST models via timestamps and versioning, but also continuous temporal changes to describe the movement or change of objects independently from their object identification. Also the integrated space-time-scale approach requires specific attention for topological relationships between (continuously) evolving geographic objects. It should be noted that temporal modelling itself also has a scale dimension, i.e. at what level of detail the time dimension (temporal resolution) is represented. This relates to temporal granularity as explained above. In our research we will pay specific attention to allow modelling multi-scale time dimension and the interplay with the multi-scale geometric dimensions.

More researchers have identified the need for a generic spatio-temporal data model. A theory on a unified spatial-temporal data model was proposed in [37] and a generic spatio-temporal data type in a relational DBMS was suggested in [38] and [39]. We will use these studies to model the time dimension with the other dimensional concepts of geo-data, also incorporating the syntax for (fuzzy) time dimension as specified by ISO for Geographic Data Files (GDF) in the transport domain [40].

From the above it becomes clear that several temporal aspects can be relevant and therefore they may question our approach to treat time as one dimension. Specifically, the bi-temporal model including both the real-world and the system time, is a good example showing that time is more than just a single dimension. However, in this paper we limit ourselves to treat only one temporal dimension in the 5D model/structure (without overlaps and gaps). Most likely this will be the system time and other temporal attributes will be treated as normal, non-integrated, attributes (for the time being). This is motivated by the assumption that for system time it is more feasible to avoid the overlaps and gaps.

2.4 Multi-scale Modelling

Modelling different scales of geo-data is related to the “coarse-to-fine” hierarchical structure of how we perceive, model and understand our environment. In some applications less detailed, but simpler data works better, especially when there is a need for an overview. In other cases very detailed data is required.

Two basic options exists for maintaining data sets of the same real world at different scales. First option is to separately maintain different databases at predefined scale-steps. This option is practiced by many National Mapping Agencies that produce maps at different scales. Second option is to maintain only the most detailed data and to automatically generalise small scale data from it on the fly, eventually supported by pre-storing the results of costly geometric computations in multi-representation (as in the first option).

To provide and reuse multi-scale data within the GII, consistency between data at different scales is fundamental, i.e. the availability of data at different scales free from contradictions enabling smoothly zooming in and out. This is supported by multi-representation data models that formally define different scale states of the data.

Many researchers have studied multi-representation data models since it was introduced in [41, 42]. Examples are MRMS [43], MADS [44], Perceptory [45], modelling multiple geometries [46], modelling scale transitions between pairs of objects [47] and modelling links between instances [48]. While these previous initiatives mainly aimed at controlling the redundancy of multi-representations and multi-scale data, our study will specifically aim at reducing redundancy to improve efficiency and to better assure consistency between different scales.

Therefore we will first extend previous initiatives on multi-scale data modelling with an explicit notion of how geo-information changes at scale transitions. With this, we will build on the semantic rich information model presented in [49] that integrates the data states at different scales and at the same time formalises semantics on scale transitions. In addition we will build on the tGAP data structure [50, 51, 52], where the data structures can be queried based on the importance value of the objects, as shown in Figure 2.

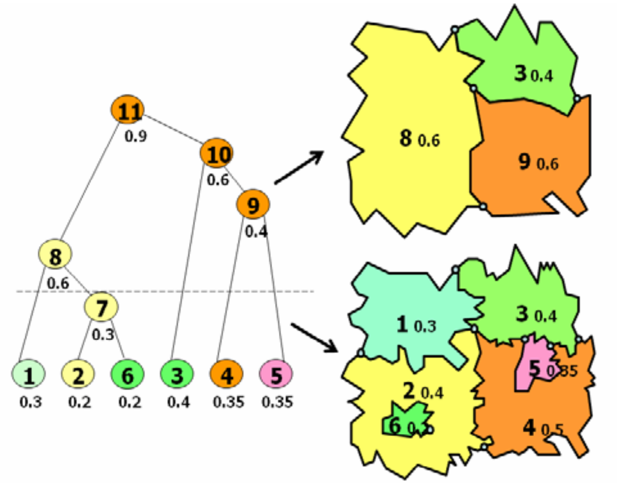

Fig. 2. Illustration of the working of the current tGAP structures

The tGAP data structure enables objects to be stored once and to be displayed at any arbitrary scale, supporting smooth zooming and progressive transfer. The idea of integrating 2Dspace and scale in the tGAP structure was first presented in [53]. A step-wise process will show how the tGAP structure can be extended to the time dimension to process time changes efficiently and to include the complete history in the same vario-scale structure. This will enable to query objects at any arbitrary scale and moment in time.

To embed the 3D scale concept in the tGAP data structure, our research will enrich the 3D Level-of-Detail (LoD) concept as studied in computer graphics with semantics on geo-information. In computer graphics LoD involves decreasing the complexity of a 3D object representation as it moves away from the viewer or according other metrics such as object importance or position.

3 Mathematical Theories on Multidimensional Modelling

To realise a 5D geo-data modelling approach by which the treatment of (up to 3D) space, time and scale is optimally integrated, we will study existing mathematical theories on multidimensional descriptions and apply them to the well defined frameworks for 3D, time and scale modelling in the geo-information domain. Examples of established mathematical theories on multidimensional modelling are:

- Topological polyhedra where multidimensional objects are built from their lower primitives, i.e. a 3D volume object consists of 2D faces that consist of 1D edges that consist of 0D nodes [54, 55].
- Regular polytopes, which is based on a division of space by hyperplanes, e.g. a 3D volume object is described by 2D planes [56].

- Simplicial Homology based n-simplexes, which are the building blocks for the Triangular Irregular Network (TIN in 2D) and Tetrahedral Network (TEN in 3D) and their higher dimensional equivalents [57].

The first theory is advantageous for multidimensional geo-data modelling because it aligns to the boundary representation of 3D volume objects of OGC [58]. However since it lacks of a well defined fundament, validity of objects has to be fully handled by additional functionalities. The advantage of the second theory is that the formalisation of multidimensional concepts is straightforward because the primitives that build an object are described with equations of the hyperplanes (and are valid in any dimension). Finally, the third theory is advantageous for geo-data modelling because of the n-simplex (e.g. triangle, tetrahedron) based approach. Triangles contain specific characteristics, such as convexity, that make it easy to enforce validity of objects that consist of the lower dimensional primitives (and again this theory is valid in any dimension).

Potentials of the three theories for handling multidimensional geo-data are shown by a) [16, 17, 18], b) [19], and c) [20] who implemented the respective three theories into a 3D data structure.

To explain how the simplicial homology theory may be applied to multidimensional geo-data modelling, we will now describe how it was applied in [20] resulting in a network of simplicial complexes forming a partition of space, i.e. a TEN in 3D (see Figure 3). The boundary of a n-simplex Sn is defined as the sum of n+1 simplexes of dimension n-1; e.g. the tetrahedron S3 (n=3) has 3+1=4 boundaries, which are of dimension 3-1=2 (triangles).

However, the theory is valid for any dimension, so S4 has 5 boundaries of dimension 3, that is 5 tetrahedrons. How would these fit in a network of simplicial complexes in 4D and how could this be used to present the tight integration of 3Dspace and time (or scale)?

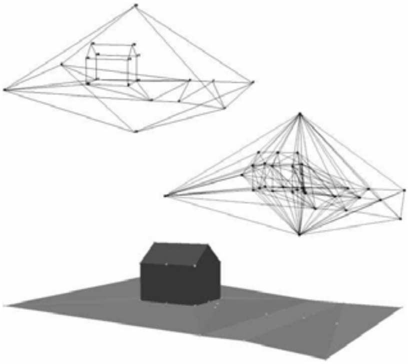

Fig. 3. Simple scene and the TEN

Our research will further study how well the different mathematical approaches are fit for 5D data modelling and also whether other mathematical approaches might better fit.

4 Research Methodology for 5D Data Modelling

Because of the unexplored domain of deeply integrated 5D information modelling, much knowledge need to be gained on the optimal 5D approach. To do so, in our methodology we propose to first apply mathematical theories on multidimensional modelling to principles established in 2D/3D, spatio-temporal and multi-scale data models and to gradually extend the results with extra dimensions in three iterations (A, B and C), see Figure 4. This will lead to three alternative 3D models in the first iteration (3Dspace, 2Dspace+time, 2Dspace+scale) and three alternative 4D models in the second iteration (3Dspace+time, 3Dspace+scale, 2Dspace+time+scale), finally leading to the best 5D data model in which lower dimensional objects are supported as well. The intermediate trajectory is important to optimally prepare the separate approaches for an integrated 5D data modelling approach and to gain fundamental knowledge on how to best address the different dimensions in the integration, both at conceptual model level and on database technology level.

The steps applied in every iteration are: 1. Conceptual modelling, 2. Implementation (with test data), and 3. Testing and validation (with real world scenarios).

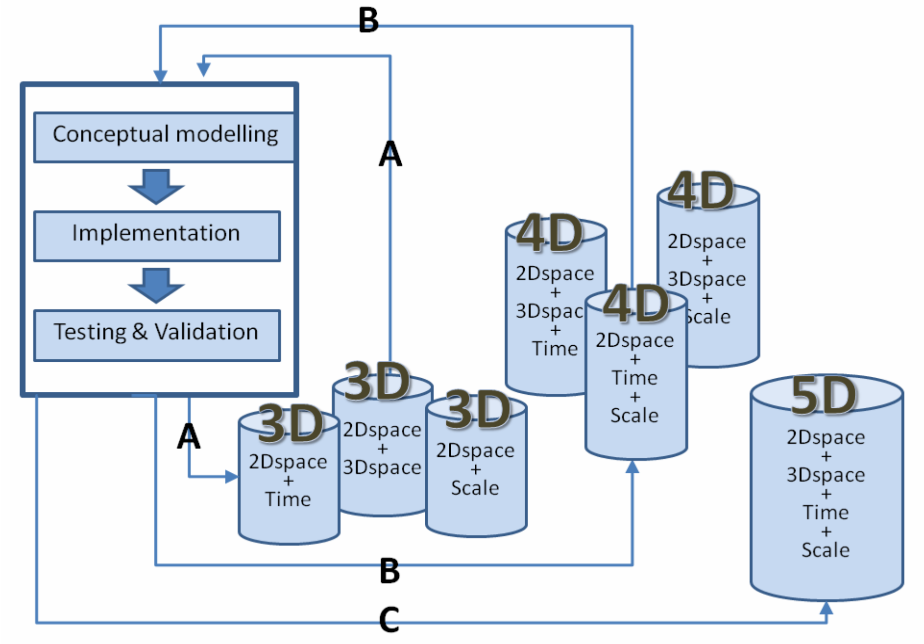

Fig. 4. Workflow of research methodology

Since *iteration A* starts from established principles, the resulting 3D models should be well feasible and in reach, also because they can be built on the partial models that are already operational, e.g. 3D in commercial systems (Bentley Systems, Oracle, ESRI), spatio-temporal databases and vario-scale data structures. Therefore the main aim of this iteration is gaining insight into how integrated spatial and time (or scale) dimensions behave when applying the multidimensional mathematical theories (simplicial homology, regular polytopes).

The combined dimensions of *iteration B* belong to hardly explored types of models and implementations, but we expect them to be feasible (in part). In addition they will provide insight in even more complicated integrated 5D modelling, which is still required as these 4D models focus on a selection of multidimensional concepts only. Again these explorations will provide more insight in the behaviours of integrated dimensions.

Iteration C will use the knowledge from iterations A and B to generate concepts for the 5D data model deeply integrating space, time and scale concepts implemented with a 5D data type, 5D topological structures and primitives as well as 5D clustering and indexing.

For validating the (intermediate) research results we will establish application tests with large datasets containing 2Dspace and 3Dspace geo-information at several scales also containing time information. For these tests we can make use of the spatio-temporal cadastral database and of large, mid- and small-scale topographic datasets of the Netherlands' Kadaster. In addition various large 3D datasets are available, such as the Actual Height Model of The Netherlands, 3D datasets of municipalities (Amsterdam, Rotterdam and Tilburg) and a 3D detailed topographic dataset of Rijkswaterstaat.

5 Discussion

This paper proposes a research methodology for 5D data modelling that fully integrates 2D/3D space, time and scale aspects of geo/information. The methodology combines established principles on 2D/3D space, time and scale modelling in the geo-information domain with mathematical theories on multidimensional modelling. Although 3D, time and scale aspects have been studied in separate research domains, studying the deep integration of time and scale concepts in the traditional 2D/3D models to replace their separate time and scale treatment with a full partition of 3Dspace+time+scale is new. Unique is also the approach to combine both fundamental theories with information technology (i.e. DBMS implementation) in multidimensional data modelling. The approach will result in a new theory and method for geo-data modelling as well as technologies realising a multidimensional partitioning.

Integrating multidimensional concepts of geo-data enables shared geometry and embedded topological, temporal and scale structures through a full partition of 3Dspace+time+scale. Instances can be identified as separate features but they will not have independent 2D/3D, scale and temporal attributes. Instead they will refer to primitives in the full multidimensional continuum. The resulting model will contain a highly formal definition of the dimensional concepts of geo-data allowing optimal flexibility to define specific semantics for each feature type and each dimension separately.

As the research approach extends currently available single-dimensional models in a step-wise approach, the intermediate models that integrate multiple but not all dimensional concepts are already in reach for use in practice and commercial implementations within the next few years, i.e. several 3D models after one year (3Dspace, 2Dspace+time, 2Dspace+scale) and several 4D models after three years (3Dspace+time, 3Dspace+scale, 2D+time+scale), all based on a solid mathematical theory.

Several stakeholders will benefit from this research. A first group of stakeholders that will benefit are providers of geo-information for which the multidimensional data types provide important advantages with respect to efficiency and consistency compared to the current separate treatment of space, time and scale. A common characteristic of these providers is that they are responsible for maintaining and providing large amounts of geo-information at different scales for which it is increasingly important to keep history track record. An integrated approach for multidimensional concepts of geo-data enables these organisations to be optimally prepared for provision of geo-information in the Semantic Web in the future. A second group of stakeholders that will benefit from the intermediate and final results are vendors of geo-ICT systems that can implement the multidimensional data types as realised in prototypes. A final group of stakeholders, and perhaps in the long term the most important group, that will benefit from this research are end-users of geo-information who will be served by improved and new 5D aware applications and services.

In the long term results on 5D data modelling in the geo-information technology domain are important for standards on geo-information which are established and developed by ISO TC 211 and the Open Geospatial Consortium.

Finally it should be noted (and it is quite well-known) that depending on the application, different types of objects may be more (or less) relevant than others. This then results in a different generalisation/scale structure. One could image having different 5D representations for different applications, all starting from the same base data. The existence and relative importance of the classes in the various applications could also be considered in a more integrated manner as the sixth dimension: the semantic-dimension. For, the time being this is considered out of our research scope (and single application profile is assumed).

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